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STUDENT VERSION

Fastest Fastball?

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Abstract: We consider the problem of comparing pitch velocities using measurement methods in different eras of baseball.

Keywords:

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INTRODUCTION AND SCENARIO DESCRIPTION

Pitch velocity is one of the most fascinating statistics in baseball, as documented in the 2015 documentary *Fastball* [3]. Modern measurements of pitch velocity are taken as the maximum velocity achieved at any point between the pitcher's hand and home plate [4]. However, the velocity of the ball slows down due to air resistance. Thus, we may assume this is just the instantaneous velocity when the ball leaves the pitcher's hand. In the past the velocity measurements were taken at specific distances from home plate. From 2006 to 2016, pitch velocity was recorded by the PITCHf/x system which measured velocity at a distance of 50.5 feet from home plate (10 feet from the mound) [6] [7].

Task 1: Assume that the velocity, v, of a pitched baseball satisfies the differential equation

$$mv' = KA^a\rho^b v^c,$$

where m is the mass of the baseball, A is the cross sectional area of the baseball, ρ is the air density, and K is dimensionless. Show that in order for this model to be dimensionally consistent, we must have

$$v' = -kv^2$$

for some constant k. Also, explain why it make sense to assume that k is positive.

Unfortunately, Major League Baseball does not seem to publish any values for k (which would vary according to a variety of factors). They do however publish a variety of other pitch statistics. Some of them are as follows:

- *Velocity (VELO):* As stated above, this is the ball's maximum velocity achieved at any point between the pitcher's hand and home plate.
- *Extension (EXT):* This is how far in front of the pitcher's mound the ball is released.

In order to determine k one needs measurements of time that the ball is in the air. The Diamond Kinetics Pitchtracker baseball [2] (a Bluetooth enabled baseball for pitch training) also measures the following:

- *Time to Plate:* This is the time it takes from the pitcher's initial movement until the ball gets to home plate, which is 60.5 feet from the pitcher's mound.
- *Delivery Time:* This is the time from the pitcher's initial movement until the ball is released.

A collection of data from one pitcher in a study of the Pitchtracker ball at [2] is given by the following table:

Velocity (mph)	Time to Plate (s)	Delivery Time (s)	Extension (ft)
85	1.2	.74	5.1
83.7	1.3	.84	6.2
79.3	1.25	.79	4.1
83.6	1.4	.95	4.8
82.3	1.21	.75	5.7
85.7	1.26	.81	5
81.7	1.17	.71	5.9
86.7	1.24	.79	4.5
77.7	1.23	.72	5.7
72.7	1.25	.74	3.2
77.3	1.37	.86	5.8
76.9	1.25	.74	5.8
73	1.22	.69	4.4
77.1	1.24	.78	6

Let's try to use this data to find k.

Task 2: Solve the differential equation you derived in Task 1 analytically for velocity. Use this to solve for displacement, s, in feet from the pitcher's mound (consider the direction toward home plate to be the forward direction). Use this to attempt to find k from any row of the above data set. Explain where and why you get stuck.

Task 3: Use numerical/graphical methods of your choice to estimate k as accurately you feel is possible from the above data set. Be sure to explain your process. Here are a few hints:

- For any row of the data set, you will need to begin with an initial guess for k and narrow it down to match the data. It's good to know in advance that k is quite small.
- Some of the data is erroneous, so you need to keep an eye out for it.
- To avoid frustration, be careful with your units(!).

Task 4: The fastest pitch velocity ever recorded was by a 105.1 mph pitch by Aroldis Chapman on September 24, 2010. This velocity was measured 50.5 feet from home plate (in the PITCHf/x era) [1]. In 1974, Nolan Ryan had his fastball velocity clocked at 100.9 mph [5] (in the 9th inning of a game he started!). This measurement was taken 10 feet in front of home plate. Use your model to compare these pitches. Which pitcher really holds the title of being the hardest thrower?

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