Article Review and Annotation


Article Abstract: A description of a course for mathematics and physics students in which they are encouraged to construct simple mathematical models of situations in biology and other sciences, as well as learning to solve the differential equations that appear in such models.

The paper is about details of a course taught by the author and of student reaction, one of which is charming,

“...most of the work in Pure Maths has been with boring, tedious definitions and proofs, whereas the interesting and fascinating side of mathematics, that exists in Penguin Paperbacks, which I was hoping for at University has been in rather short supply. [ p. 469]

To which the author replies simply in typically English understatement, “This last complaint deserves some attention.” [p.469]

The very useful portion of the article is the appendix, entitles “Sample problems,” and contains statements with references for 15 problems, several of which we literally lift here.

(1) Write down a differential equation modelling population growth when the maximum population that can be supported by the environment increases steadily with time (due, for example, to improvements in farming technique).

(2) Modify the logistic model to include time-lags: the birth-rate at time \( t \) might depend not only on the population at \( t \) but also on the population at time \( t - c \), where \( c \) is the gestation period. This gives a differential-difference equation, not a differential equation.

(3) Modify the simple Volterra equations to include (a) the fact that the prey would not increase indefinitely even if there were no predators, and (b) the fact that the predators could survive on other food even if the prey were absent.
(5) The economics of a single commodity depends on three quantities: the supply $S$, the demand $D$, and the price $P$. It is reasonable to suppose that the price rises when demand exceeds supply, that the supply depends on the price and increases with price, and that demand depends on the price, and may also depend on the direction of price movement (i.e., there may be an extra incentive to buy now if one thinks that the price will rise again soon). Construct a mathematical model along these lines, and obtain a differential equation for $P(t)$ which does not involve $S(t)$ and $D(t)$.

(6) A certain model of social interaction in small groups of people works with four variables: $I$, the amount of interaction between members; $F$, the level of friendliness; $A$, the amount of activity carried out by members; and $E$, the degree of external pressure on the group—more precisely, the degree of activity imposed on the group from outside (it may, for example, be required to produce a certain amount of goods). It is reasonable to suppose that a friendly group will generate more activity and interaction. Try to formulate other relations between $I$, $F$, $E$, $A$, and thus set up a system of equations. (See Chap. 4 of *Models of Man*, by H. A. Simon.)

(11) If $P$ parasites are living on $H$ hosts, one might expect that the growth of $P$ will be controlled by $P/H$, the number of parasites on each host, so that $P/H$ will approach some equilibrium value. Formulate a suitable modification of Volterra’s equations, and study its behaviour in the phase plane.

We conclude by quoting (extensively) the author’s opening remarks which (unfortunately) are still timely.
R. L. E. Schwarzenberger, in the introduction to his book [6], makes the point that very many students are quite adept at manipulating symbols according to the rules of algebra and calculus, but quite bad at understanding what the symbols mean. For example, they will happily differentiate the function \( \log(\log(x)) \), without realizing that (as a real-valued function) it does not exist for any \( x \); and many students will produce without comment an obviously wrong answer to, say, a mechanics problem, when a moment's thought (taking a simple special case, perhaps) would show that it cannot be correct. Schwarzenberger takes a geometrical and graphical approach to differential equations, and this is a valuable treatment for the symptoms described above—trying to sketch the graph of the function \( \log(\log(\cos x)) \) soon reveals that it does not (really) exist, let alone have a derivative. But the exercise of constructing mathematical models of, say, a population growth process, and then interpreting the mathematical results in terms of the behaviour of the population, is also good training in understanding mathematical formulae, in getting into the habit, for example, of noticing whether functions are increasing or decreasing, and whether it is reasonable that they should do so; in short, learning to speak the language of mathematics fluently and intelligently.

For the purposes of this course, then, the principal aim in studying modelling is not to gain an appreciation of the role of mathematics in science, nor the nature of applied mathematics (though this is a valuable subsidiary aim); it is to improve the general mathematical skill of the students, and develop a feeling for the subject, especially in those to whom it does not come naturally. This aim will

For completeness we offer the author's references.

References