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STUDENT VERSION
Heart Death Rate

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SCENARIO DESCRIPTION

Heart disease is the leading cause of death in the United States. It is not a single disease. Rather, a collection of diseases of the heart and blood vessels collectively known as cardiovascular disease. Since 1950, death rates from heart disease and stroke have declined sharply representing one of the most important health achievements of the 20^{th} century. Intensive investigations led to finding major risks factors of the disease are cigarette smoking, high blood pressure, elevated total cholesterol, obesity, diabetes, and physical inactivity. Subsequent development in early detection, prevention, and treatment of these risks factors due to medical care have been very effective in decreasing the heart death rate. In the 21st century, still heart disease is the leading cause of death though it is following the decreasing trend. We received the following data set to explore the mathematical modeling opportunity.

Data set: This example data is taken from [1, Ch 4, Sec 4.2, Exercise 11, pp 198], Health: The table shows the age-adjusted death rates per 100,000 American for heart disease. (Data from: U.S. Center for Health Statistics.)

In-Class Activity

We found the data as an exercise in [1]. The data displays a downward trend. Although death from heart diseases is the leading cause of death in the United States, the decreasing pattern is encouraging to model it mathematically. We propose to develop three modeling activities for the same data set as an individual class activity. Therefore, our priority is to model it initially as an algebraic exponential decay and difference equation method, and finally as a differential equation to predict the downturn disposition.

Year	Time	Death Rate
2000	0	257.6
2002	2	240.8
2004	4	217
2006	6	210.2
2008	8	186.5
2010	10	178.5

Table 1. Heart death rate between the years 2000 and 2010.

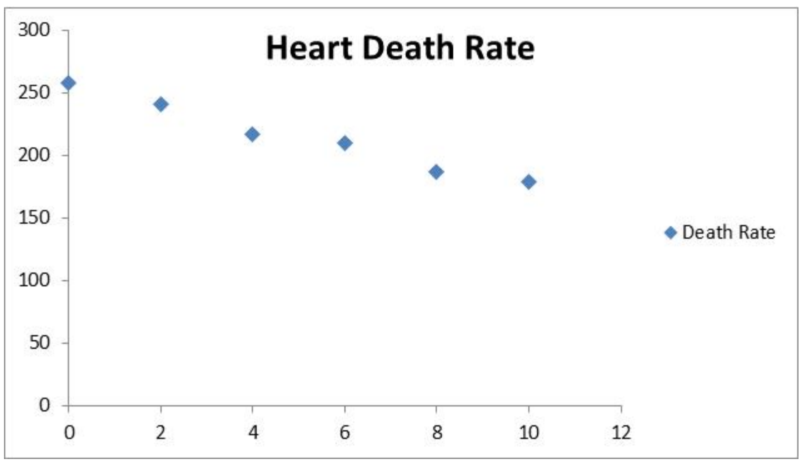

Figure 1. Plot of the data from Table 1 in which x-axis represents the time in years and y-axis represents the death rate for heart disease.

(a) Exponential Function Model

Consider the data set given in Table 1. Assume that the year 2000 represents the initial time 0 for data collection. Further, it shows that data has been collected in every two years and reported under the column death rate. Insert the data on a spreadsheet and plot a scatter plot. Analyze the graph and discuss your observations.

Build a model for $H(t)$, the heart death rate after t units of time, where $H(0) = 257.6$. State your assumptions and variables clearly and show how they are used in the modeling process to obtain an algebraic equation. Compare your model predictions with the data and graph your results on the same plot.

(b) Difference Equation Model

From Figure 1, observe that the death rate is decreasing. So we want to model it mathematically by comparing the consecutive terms and recognizing a pattern between them. In order to do that first state your assumptions and variables clearly to write heart death rate in discrete notation. If H_n represents the death rate at time step n, then H_{n+1} represents the future value. The goal is to determine H_{n+1} from the previous discrete value H_n as $H_{n+1} = f(H_n)$. To find the relationship between two consecutive terms, one can write either the difference or ratio between the terms as $H_{n+1} - H_n$ or H_{n+1}/H_n, respectively.

Investigate the difference and ratio of the heart death rate of consecutive terms. Display your findings on a spreadsheet in column form and analize your results to draw your conclusions. Use the ratio pattern to write a mathematical model. Compare your model predictions with the data and display the graphs on the same plot.

(c) Differential Equation Model

From the scatter plot Figure 1, observe that the death rate is decreasing. Build a model with differential equation as a continuous death rate for the heart disease after t units of time. Assume that the rate of change of the heart death rate is directly proportional to the size of the heart death rate. Investigate the choice of the proportionality constant as a decay model with negative value. Hence, build a model for a parameter estimation for the differential equation. Using EXCEL spreadsheet, guess the value of the constant by hit and trial method to find an admissible value of the parameter. Compare your model predictions with the data and show your results with graphs.

(d) Parameter Estimation

Guessing a value of the constant or parameter is an *ad hoc* process. The procedure is time consuming and inaccurate. In order to get the best fit, one can use a parameter estimation procedure using EXCEL spreadsheet. Use the EXCEL built-in command 'solve' to find the best parameter automatically for the differential equations modeling. Similarly, investigate the parameter estimation procedure for the algebraic exponential decay and difference method. At the end, compare all three methods with the original data and visualize the graphs on the same plot.

REFERENCES

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