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STUDENT VERSION

Risk of Infection?

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Abstract: This project is designed to examine differences between the exponential and logistic growth models in biology and how to apply these models in solving epidemic questions. This project was designed for an introductory section in Calculus II or a course involving ordinary differential equations, which are appropriate for second year undergraduate students majoring in mathematics, physics, and bio-mathematics. This project provides an opportunity for students to develop their understanding of differential equations and increase their appreciation of mathematics as it applies to solving a problem of biology.

Keywords: infection, carrying capacity, exponential model, logistic equation, COVID-19

Tags: Calculus II, differential equation, first-order, nonlinear

SCENARIO DESCRIPTION

Part I

He immediately noticed that most people were wearing a face mask. There were big signs everywhere stating: “Stay Away from Camels and Be Aware of MERS”.

James had never heard of this before, so he decided to do some research while he was waiting for his flight. Using his smartphone, he easily found an article about MERS on the CNN Website. He found it is a virus and MERS stands for Middle Eastern Respiratory Syndrome. He read: “Starting from May 20th, 2015, there was breaking news in South Korea about Middle East Respiratory Syndrome, which was originally discovered in Saudi Arabia (hence the name). On June 7th, 2015, South Korea reported its fifth death from MERS, bringing the total number of confirmed cases to 64. 1,820 people remain quarantined, either at home or in health facilities. And more than 1,300 schools remain closed.”(1) James had also seen Ebola in the news a few years ago, which was really deadly to humans too. From times.com, James found out that MERS doesn’t appear to be able to spread as quickly as Ebola can (2). While Ebola spreads through direct contact with the bodily fluids of an infected person, MERS doesn’t spread easily from person to person, and though it can spread through the respiratory tract, very close contact is needed.

James recalled that there was an outbreak of flu on the campus of Vanderbilt University in the winter of 2013. The campus had to close for three days. And lately, Ebola has been in the news. So what's the difference between MERS, Ebola and the flu anyway? So he went back to his notes and constructed a table to show the major traits for each virus (Figure 3). He began to wonder whether a college student in the U.S. was at risk for getting Ebola. Since he was just in South Korea, another worry was whether he might have been exposed to MERS. "And what does disease "spreading" really mean?"

Back at Vanderbilt University, James went to his virology instructor, Dr. McCarroll, for explanations. Dr. McCarroll told him that there are no available vaccines to control either Ebola or MERS, but there was a vaccine to control the spread of flu. It's made available after the first reported outbreak of the year. As a result, the prevalence of Ebola and MERS could continue to grow over time, while the flu virus would eventually reach a maximum point and become stable. This is why the people who get Ebola and MERS need to be immediately isolated for medical treatment.

James found a very informative website from the New York Times: http://www.nytimes.com/interactive/2014/07/31/world/africa/ebola-virus-outbreak-qa.html#origination . As James read about Ebola, he was surprised to learn that it is a zoonotic disease. This means it is transmitted across species. Ebola was discovered in 1976 and was once thought to originate in gorillas, because human outbreaks began after people ate gorilla meat. Scientists now believe that bats are the natural reservoir for the virus, and that apes and humans catch it from eating food that bats have drooled or defecated on, or by coming in contact with surfaces covered in infected bat droppings and then touching their eyes or mouths. The current outbreak seems to have started in a village near Guéckédou, Guinea, where bat hunting is common, according to Doctors without Borders.

How Does the Disease Progress? Symptoms usually begin about eight to 10 days after exposure to the virus, but can appear as late as 21 days after exposure, according to the C.D.C. At first, it seems much like the flu: a headache, fever and aches and pains. Sometimes there is also a rash. Diarrhea and vomiting follow. Then, in about half of the cases, Ebola takes a severe turn, causing victims to hemorrhage. They may vomit blood or pass it in urine, or bleed under the skin or from their eyes or mouths. But bleeding is not usually what kills patients. Rather, blood vessels deep in the body begin leaking fluid, causing blood pressure to plummet so low that the heart, kidneys, liver and other organs begin to fail.

Questions:

(1) Using external resources to explain the difference between infection, transmission and virulence.

(2) Using external sources, describe what positive or negative-sense ssRNA means with regards to virus replication? How does having positive or negative ssRNA make a difference with respect to infection, transmission or virulence?

(3) How can Ebola be transmitted? What makes a virus an “airborne” virus?

(4) What kinds of symptoms do flu victims suffer than Ebola victims generally do not?

Part II

Dr. McCarroll reminded him that she had discussed two growth models in her virology class that were based on differential equations. Since James was in ordinary differential equations, he decided to do a little reading ahead on the exponential growth and logistics growth models. He learned that these could be used to predict the changes of the number of infected people if the spreading rate of the viruses were provided.

The exponential growth model is defined as $dN/dt=rN$ and the logistic growth model is $dN/dt=rN(1-N/K)$, where N is the number of infected people with this virus, r is the per capita infection rate for people who are exposed to this virus, and K is the maximum number of people in the population that could be infected by this virus, (i.e., the carrying capacity of the population in this model). The exponential growth model can be used to measure the population that continues to grow forever and the logistic growth model can be used to measure the population that will reach a maximum (carrying capacity) after some time.

Questions:

(5) Solve and graph the solution of the exponential growth model with the fixed per capita growth rate $r = 0.1$ and the initial number of infected people: $N(0) = 10$.

(6) Solve and graph the solution of the logistic growth model with the fixed per capita growth rate $r = 0.4$ and the initial number of infected people: $N(0) = 1$, and the carrying capacity $K = 5000$.

(7) What mathematical principles describe the spread of disease?

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