

## STUDENT VERSION

### Vibration Analysis Based on Half-Car Model

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**Abstract:** Vibration vehicle models provide a great opportunity to integrate vehicle-based vibrations into a mechanical engineering vibrations course. This project works on a multiple-degree-of-freedom (MDOF) including the pitch and bounce of the vehicle body mounting on a suspension system. The approach utilizes differential equation (DE) based mathematical modelling to solve speedbump induced vibration. The problem involves a half-car approach to a vehicle's response when subject to a speed bump, or a sudden impulse load upon the model. These responses are captured and analyzed on pitch and bounce of the model at the center-of-gravity of the model. Finite element analysis (FEA) will be used in this project.

Students need be familiar with planar kinematic and planar kinetic analysis of rigid bodies, Laplace transform and inverse Laplace transform, and a knowledge of fundamental concepts in vibration analysis. It is suggested that each group is composed of three students. It will take students 5 weeks to complete this project, including their in-class oral presentations.

**Keywords:** differential equations, Laplace transform, vehicle vibration analysis, half-car model, finite element analysis

**Tags:** shock absorbers, suspension, system

## SCENARIO DESCRIPTION

### Background

A systematic treatment of a vehicle as a vibration system best starts with the basic properties of a vehicle on its suspension system – i.e. the motions of the body and axle. The body, which is considered to be the sprung mass portion of the vehicle, moves as an integral unit on the suspensions. The axles and associated wheel hardware, which form the unsprung masses, also move as rigid bodies and consequently impose excitation

forces on the sprung mass. The vibration behavior of a vehicle can be characterized most meaningfully by considering the input-output relationships. The input may be any of excitations (impulsive input when hitting a speed bump in this project, for example) and the output most commonly of interest will be the vibrations of the body. At the most basic level, all vehicles share the absorption property to the sprung mass supported by primary suspension systems at each wheel.

Vehicle vibration analysis in this project is based on half-car model, as shown in the following figure, in which it is assumed that the vibrations are in symmetry along the car's long axis (relative right-side vibrations are equal to the relative left-side vibrations). The half-car model will be used in calculating the bounce and pitch responses of the car body when subject to speed bump which is represented by impulsive inputs on the front wheel and rear wheels.

Write a report to document your project work. The project report should include (i) a brief introduction and statement of objectives, (ii) a complete sufficiency analysis and (iii) a discussion of your results.

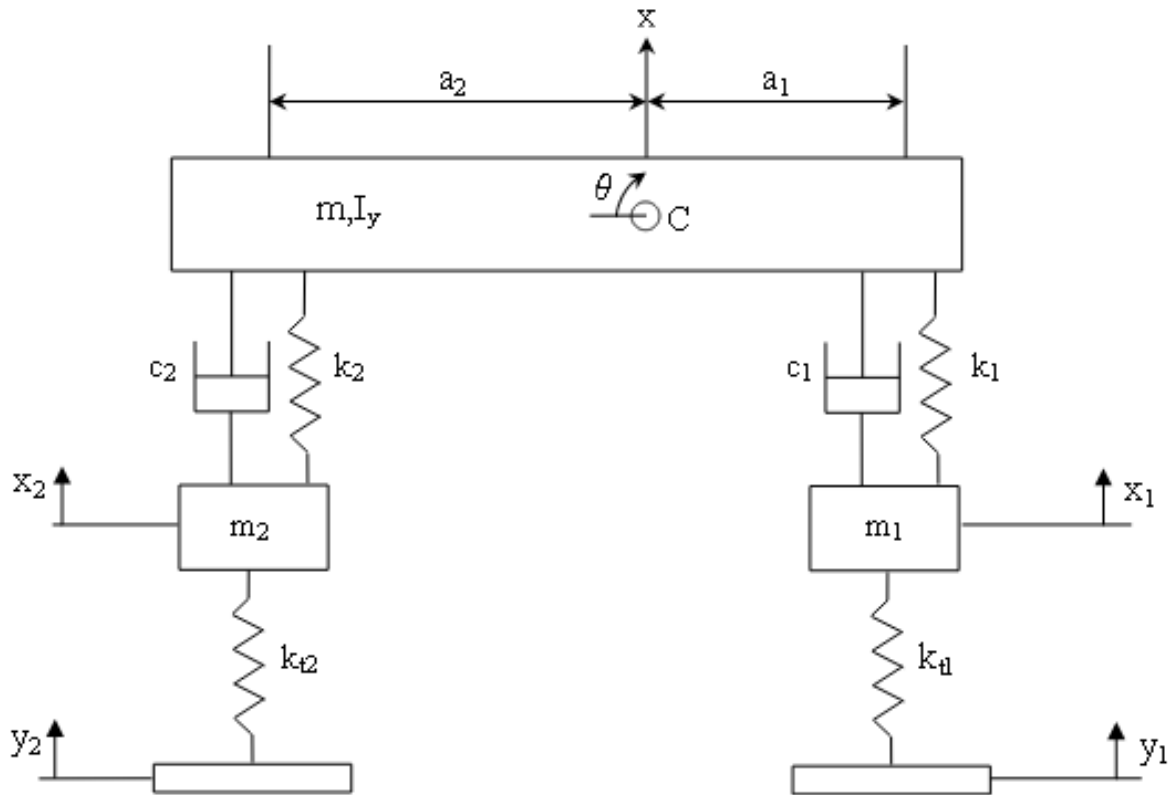


Figure 1. Half-car model for vibration analysis

## Model Development

### Stage 1: Data collection

1. Choose the vehicle model which you are going to investigate.
2. Find out the specifications of the selected vehicle, and determine all required data for this project:
  - Front and rear tire stiffness ( $k_{t1}$  and  $k_{t2}$ ).
  - Front and rear unsprung masses ( $m_1$  and  $m_2$ ).
  - Front and rear suspension parameters (stiffness  $k_1$  and  $k_2$ , damping coefficient  $c_1$  and  $c_2$ , respectively).
  - Body mass ( $m$ ).
  - Center of gravity of the body ( $a_1$  and  $a_2$  from the gravity center for the front and rear axle).
  - Mass moment of inertia of the body ( $I_y$ ).

### Stage 2: Draw free body diagrams (FBDs) and build corresponding equations of motion

1. Unsprung mass on the front side
  - Draw its free body diagram with only one translational degree of freedom (DOF) along the vertical direction.
  - Build its equation of motion based on Newton's Second Law of Motion, which must be a second-order differential equation including all parameters shown in Figure 1 in the following generic format:
$$F_1(y_1, y_2, k_{t1}, k_{t2}, m_1, m_2, k_1, k_2, c_1, c_2, m, I_y, a_1, a_2) = 0. \quad (1)$$
2. Unsprung mass on the rear side

Repeat the above analysis for the rear unsprung mass and get its equation of motion as

$$F_2(y_1, y_2, k_{t1}, k_{t2}, m_1, m_2, k_1, k_2, c_1, c_2, m, I_y, a_1, a_2) = 0. \quad (2)$$
3. Sprung mass
  - Draw its free body diagram with two DOFs: one is the up-and-down motion at the mass center (bounce motion), and the other is rotational motion about the mass center (pitch motion).
  - Build its equations of motion based on Newton's Second Law of Motion, which must be two second-order differential equations including all parameters shown in Figure 1 in the following generic format:
$$F_3(y_1, y_2, k_{t1}, k_{t2}, m_1, m_2, k_1, k_2, c_1, c_2, m, I_y, a_1, a_2) = 0. \quad (3)$$
$$F_4(y_1, y_2, k_{t1}, k_{t2}, m_1, m_2, k_1, k_2, c_1, c_2, m, I_y, a_1, a_2) = 0. \quad (4)$$
where the function  $F_3$  is based on the up-and-down motion of the body while  $F_4$  is based on its pitch motion.

### Stage 3: Laplace transform

- Take Laplace transform of each of these four differential equations.
- Rearrange the resulting four equations and express the outputs (bounce response  $X(s)$  and pitch response  $\Theta(s)$ ) as functions of inputs ( $Y_1(s)$  and  $Y_2(s)$ ) by deleting the responses of two unsprung masses ( $X_1(s)$  and  $X_2(s)$ ) in the  $s$  domain. The two excitation inputs ( $y_1$  and  $y_2$ ) are selected to be unit impulse acting on the front and rear axle, respectively.

#### Stage 4: Inverse Laplace transform

Take inverse Laplace transform and express the bounce response  $x(t)$  and pitch response  $\theta(t)$  in the time domain, which provide designers an insight of the effect of the vehicle configurations (for example, different suspension) on its vibration responses.

#### Stage 5: Result plots

Plot bounce response  $x(t)$  and pitch response  $\theta(t)$  with respect to time ( $t$ ) based on the values of the parameters of your selected vehicle model.

#### Analysis

Study the effects of changing suspension parameters ( $k_1$ ,  $k_2$ ,  $c_1$ ,  $c_2$ ) on the bounce response  $x(t)$  and pitch response  $\theta(t)$ .

#### In-Class Oral Presentation

Grading Criteria is based on the following aspects:

- The presenter is knowledgeable about the topic as evidenced by the depth and completeness of the presentation.
- The presentation is organized in a logical sequence.
- The presenter is professional during presentation.
- The presenter responds appropriately to audience questions.

#### References

1. Inman, D.L. 2014. *Engineering Vibration, 4th Edition*. Englewood Cliffs NJ: Prentice Hall.  
(Read the contents in Chapter 3 and get yourself familiar with impulsive responses of a vibrating system.)
2. Zill, D., and W. Wright. 2012. *Advanced Engineering Mathematics, 6th Edition*. Burlington MA: Jones & Bartlett Learning.  
(Read the contents in Chapter 4 and get yourself familiar with Laplace transform, inverse Laplace transform, and Laplace transform of a Dirac Delta function.)
3. Hibbeler, R.C. 2016. *Engineering Mechanics: Dynamics, 14<sup>th</sup> Edition*. Hoboken NJ: Pearson.  
(Read the contents in Chapters 16 and 17 and get yourself familiar with planar kinematic and planar kinetic analysis of a rigid body.)