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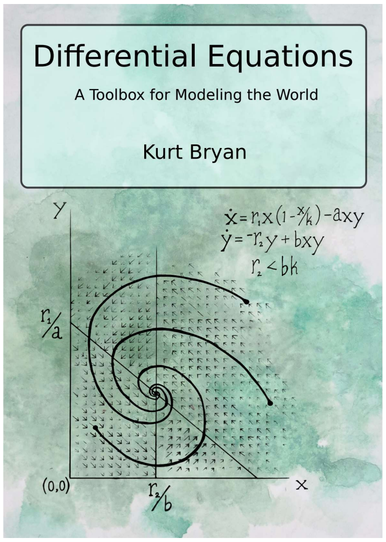
Bryan, Kurt. 2021. *Differential Equations: A Toolbox for Modeling the World*. 463 pp; $39(PDF). Cornwall, NY: SIMIODE. Available free for consideration of adoption for use in teaching, from `Director@simiode.org`. ISBN 978-1-63877-937-7.[1]



The majority of students in a differential equations class are majoring in science or engineering, rather than mathematics, and will be studying differential equations because they arise in mathematical models in their primary discipline. The departments in which they major want them to learn about differential equations in a modeling context. The mathematics faculty who want to accommodate these needs can choose a differential equations book that includes the word "modeling" in the title. Doing so does not necessarily make much difference. At worst, these books give only lip service to modeling, focusing exclusively on "applications." At best, they contain a limited amount of modeling as an add-on. Enter *Differential Equations: A Toolbox for Modeling the World* by Kurt Bryan. This book is the only one that this reviewer is aware of that presents differential equations in a modeling context rather than merely adding a bit of modeling to the standard presentation. To understand this claim, a few words need to be said about what modeling is and what it isn't. Tying a differential equation to a narrative is not modeling. Using the solution of a differential equation to address limited questions is a limited sort of modeling. Full-fledged modeling has three central characteristics:

1. Careful development of a model from assumptions about a real-world setting;
2. Investigation of whether the results are reasonable in the context of the setting;
3. Questions about how parameters affect model outcomes.

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[1] This review is based on a preliminary version of the book. The published version may differ slightly.

The second is particularly important. In mathematics, we study a world that we create because we find it interesting. The rules of that world (the hypotheses) are taken as true, and the conclusions that follow are therefore true as well. In modeling, we study a world that we create in the hope that what we learn from it will be useful. The conclusions still follow from the hypotheses, but they are only as useful as the hypotheses were good. Good mathematics applied to a bad model yields bad science. So we have to look at results differently. We don't say "The Lotka-Volterra model shows that predators cannot be hunted to extinction, therefore that cannot happen." We say "The Lotka-Volterra model shows that predators cannot be hunted to extinction, therefore the model is bad."

The first two sections set the tone for the whole book. In them, Bryan carefully develops differential equations for the time-dependent velocity of a sprinter and the amount of drug present through delivery in the inner ear. These two sections serve as models for the construction of models. The careful development is supplemented by 14 reading exercises that ask students to delve into details about initial conditions, to verify solution formulas, to elaborate on concepts, and the like. The punchline comes at the end of the second section: aside from notational differences, the two scenarios are modeled by the same differential equation. This is the ideal way to teach students the value of abstraction and generalization. Differences in typical parameter values could lead to very different typical behaviors; however, anything we learn about one of the models is immediately true for the other. After another example, the chapter concludes with a standard presentation of basic terminology and notation and a whole section on dimensional analysis. This opening chapter tells the reader everything (s)he needs to know about the aim of the book. If you want to study differential equations as a purely mathematical topic, this is not the book for you. If you want to study the mathematics of differential equations in a modeling context, you are in the right place. The book is going to include basic mathematical definitions and theory, but these will be motivated by models and scientific questions, and it is also going to include modeling topics, such as dimensional analysis and scaling, that seldom appear in a differential equations book.

Another place where the focus on modeling really matters is the chapter on the Laplace transform. Typically, authors include discontinuous and impulsive forcing at the end of their Laplace transform chapter. These topics may seem like an add-on to mathematicians, many of whom skip this material; however, the whole point of the Laplace transform is that it provides ways to solve problems with impulsive forcing and to streamline the solution of problems with discontinuous forcing. In keeping with a true focus on modeling, rather than merely a stated focus, the Laplace transform chapter in this book begins with a section that introduces the need for the Laplace transform by developing models with discontinuous and impulsive forcing.

Differential equations books range from a traditional focus on theory and analytical methods to a focus on dynamical systems and other more modern topics with only minimal coverage of the traditional topics. This book takes a balanced approach. The theory and principal methods for separable equations, linear equations, and systems are given a comparable treatment to a more traditional book, albeit in a modeling context. Relatively obscure methods, such as exact equations, and those of limited practical value, such as power series methods, are not included. Instead there are thorough treatments of numerical methods, graphical analysis, and linearized stability analysis. The chapter on numerical methods is a highlight of the book, with discussions of error as well as motivation and presentation of methods. Given that most differential equations that arise in mathematical models cannot be solved by hand, the instructor using this book should consider spending more time on numerical methods and less on the technical details of standard analytical techniques, such as problems that require specialized integration techniques or generalized eigenvectors.

Modeling with parameters suffuses the book. This can overwhelm students, who prefer numbers to symbols, but it is necessary to address important modeling questions, such as the effect a parameter has on model behavior. Dependence of outcomes on parameters appears first in a section on bifurcation in the chapter on first order equations, and it recurs elsewhere. This poses a challenge for the instructor. The text presupposes that students understand that parameters are quantities that are fixed for an instance of the model but can be changed to make different instances of the model, allowing questions such as "How does the time needed for a 100 m run depend on the runner's maximum propulsive effort?" This level of modeling carries with it an expectation of a sort of mathematical maturity that most students lack. An instructor would do well to spend some time developing the concept of parameters in an algebraic setting before diving into the first chapter.

A major pedagogical challenge in writing a differential equations book is how to incorporate the necessary material from matrix algebra. Ideally, every differential equations course would have a matrix algebra prerequisite. In practice, many mathematics curricula place the differential equations course ahead of the matrix algebra course to conform to the wishes of engineering and science departments that require their students to take differential equations and not matrix algebra. Since we can't compel engineering departments to require both courses, the differential equations author has to make a choice whether to write for an audience with a matrix algebra background or without one and then attempt to accommodate the other group. Bryan's choice is to present a treatment of linear systems that is intensive in its use of matrix algebra, including analytical solutions for complex and repeated eigenvalues, nonhomogeneous systems, and the matrix exponential. For courses without a matrix algebra prerequisite, there is an appendix on matrix algebra, but a single appendix is hardly a substitute for a whole course. The instructor who does not want to get bogged down by all that mathematical formalism and focus on analytical solutions can instead choose to limit coverage of the linear systems chapter to just that material needed to prepare students for the following chapter on nonlinear systems. In nonlinear systems, there is no need for eigenvectors or solution formulas; you only need to be able to find eigenvalues and have a theorem that connects stability to eigenvalues. A course focused more on modeling than methods should try to do as much of the nonlinear system chapter as possible while doing only those parts of the linear systems chapter that are needed for background.

Instructors who want to teach a modeling-oriented course but lack personal experience in modeling may find aspects of this book intimidating. Such instructors can get help by connecting with the SIMIODE community (https://www.simiode.org/), where they will find like-minded instructors with varying levels of experience.

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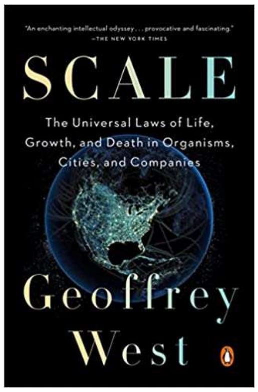
West, Geoffrey. 2019. *Scale: The Universal Laws of Growth, Innovation, Sustainability, and the Pace of Life in Organisms, Cities, Economies, and Companies*. iv + 481 pp; $30, $17.50(P). New York: Penguin Press. ISBN 978-1-59420-558-3, 978-0-14311090-3.



Curiosity is perhaps the greatest gateway for many of us to dive deeper into science and mathematics. Such amazing observations as an ant carrying 50 times its own body weight or running at a speed of 800 of its own body lengths per minute are mind-boggling, especially in comparison to our own human experiences. Naturally, we might wonder what's stopping us from carrying 50 other people at once or running a mile in just over a minute? Questions like this get me all jazzed up, and in particular, thinking about how performance scales with relative size among all living things.

Organisms seem to have natural limitations in their abilities to perform certain tasks, though, when comparing many organisms against each other, incredible relationships may emerge between performance and organismal sizes, shapes, and/or mass. Patterns appear even in how organs among

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