Adventures in Co-Teaching a Data Driven Modeling Course

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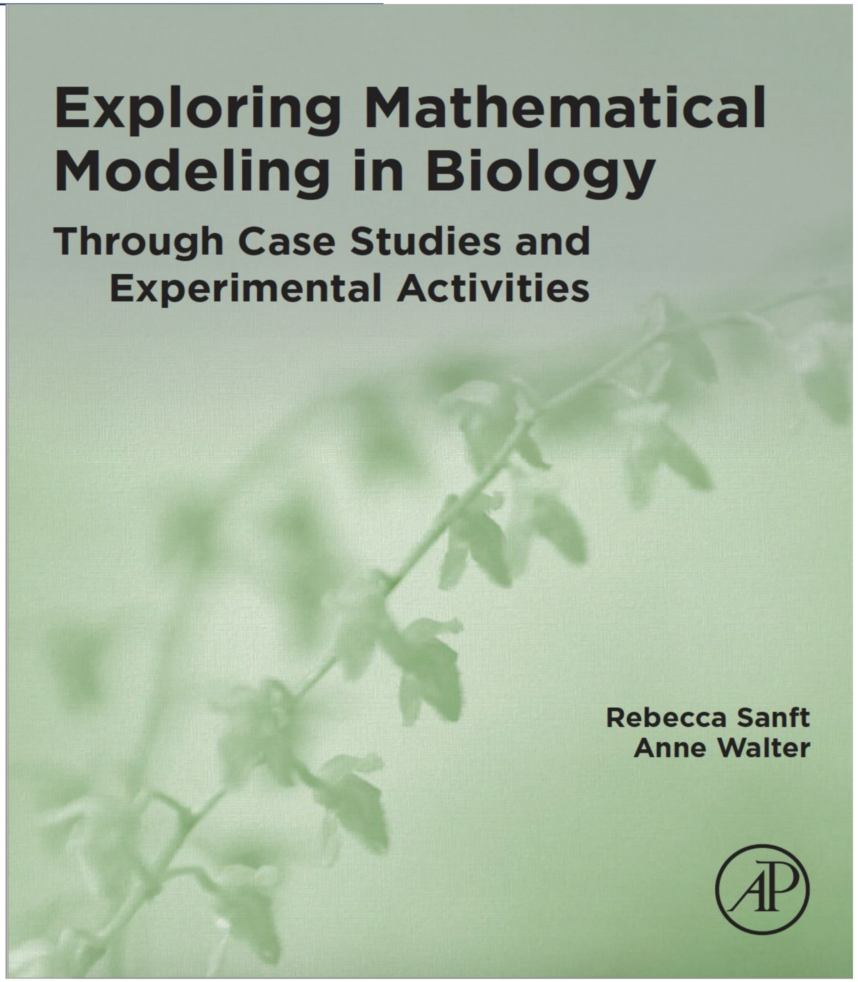
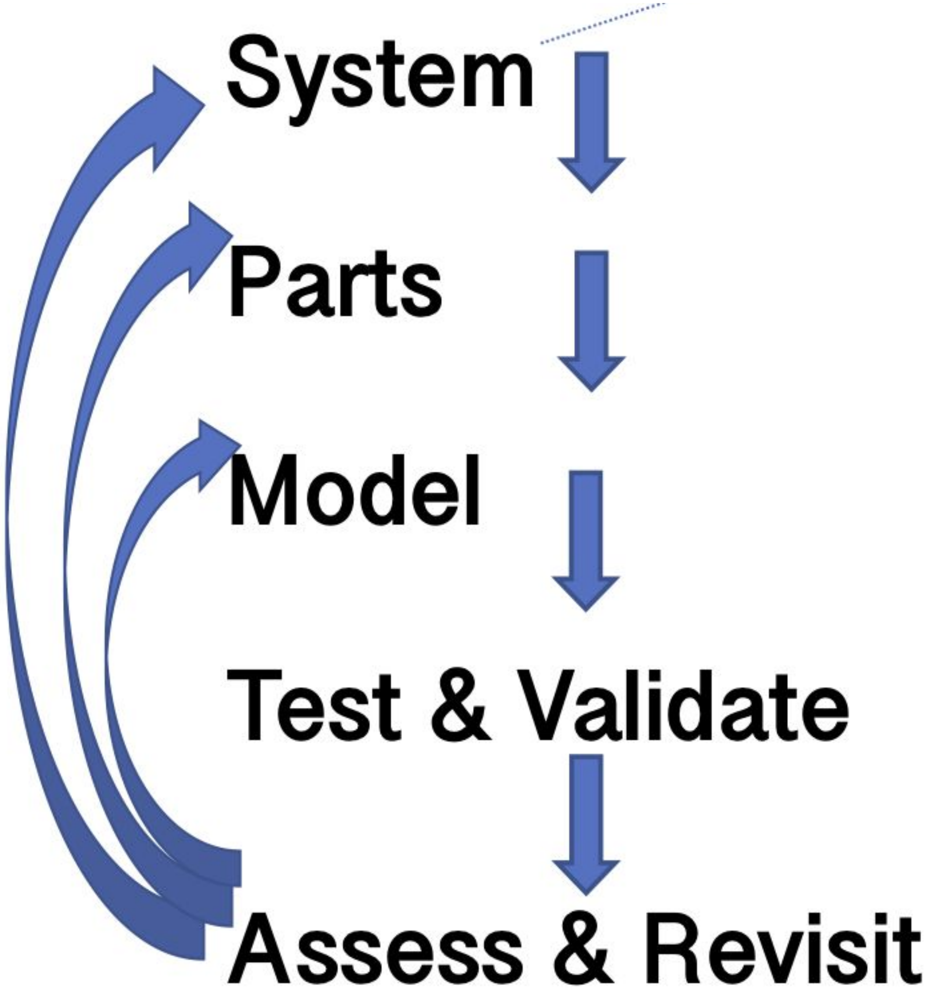


Integrate modeling steps with data collection in the lab or literature to address real questions with opportunities to assess & revisit each step.



Audience: Ours and those we can envision

Audience

- Undergraduate biology and math students with at least a minimal math background (e.g., Calculus I).
- Faculty teaching undergrads in STEM

Applicable

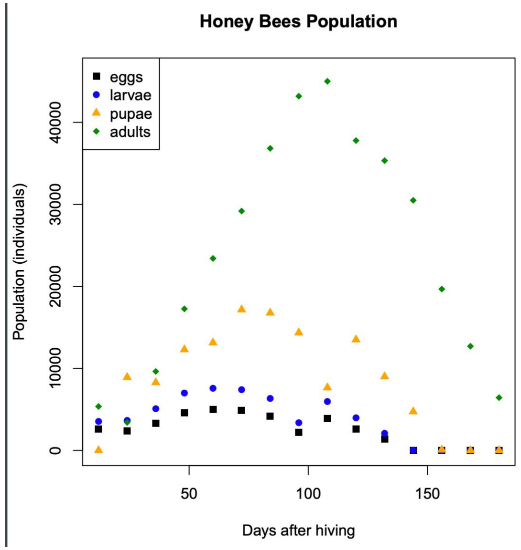
- ○ Beginning modeling courses, especially those intended for students from multiple disciplines ± lab
- ○ Supplement math or life sciences courses with individual cases & labs
- ○ Any class introducing R will find R tutorial helpful

Ideal support includes a peer teach assistant for lab & an R-tutor.

Unit 1: Preliminaries: Models, R and Lab Techniques

Practice:

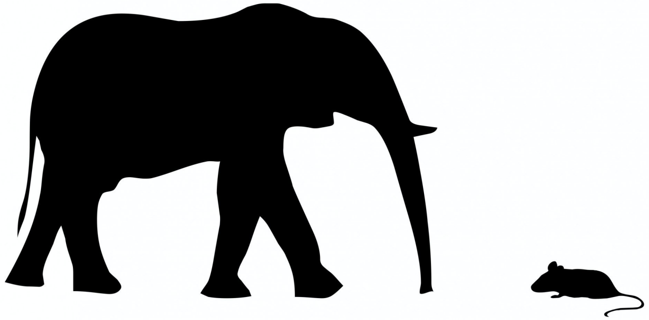
- Matrices
- Data Frames
- Plotting
- Data Files
- Iteration
- Fitting linear regression & residuals



(all exercises with biological data)

Putting it together:

- Life expectancy vs. birth year
- Allometric scaling using linearized power law for metabolic rate vs. body mass
- Another power law: species vs. area curves for bird species on West Indian Islands.

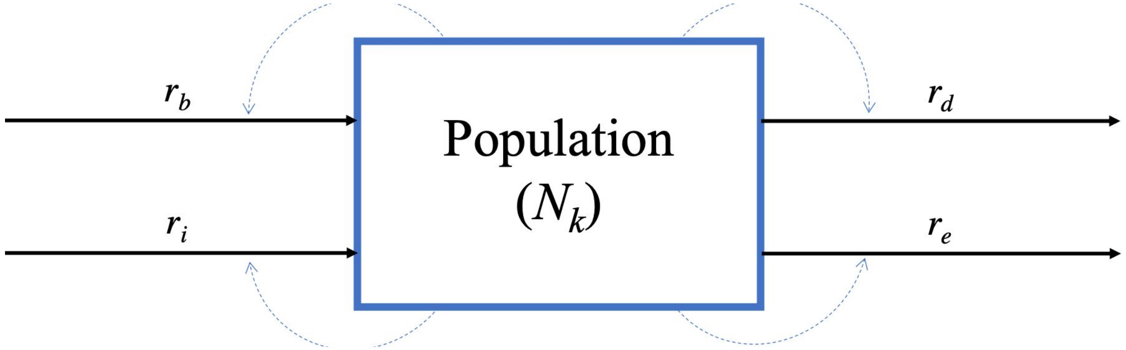


https://www.publicdomainpictures.net/en/view-image.php?image=96871&picture=elephant-and-mouse-silhouette

Pre-lab: Practicing the Fundamentals

- Pipets
- Serial dilutions
- Spectrophotometry
- Plotting data to ask different questions:
- Absorbance vs. dilution step (exponential)
- Absorbance vs. concentration (linear)

Unit 2: Introduction to Modeling Using Difference Equations



Modeling process

- Articulate assumptions
- Define questions
- Units
- Discrete-time equations
- Linear regression

Practice

- Develop models with difference equations
- Find solutions
- Population growth models of increasing complexity
- Calibrate by linear regression

Putting it all together:

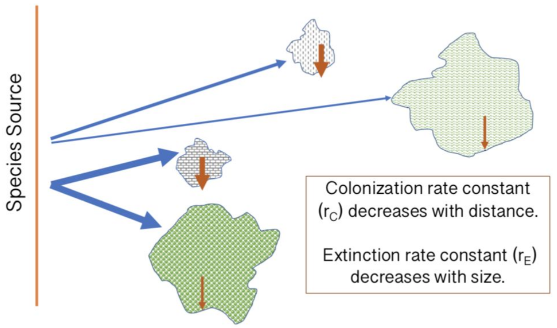
When will the whooping crane population reach a sustainable size e.g., 1000 individuals?

Given annual crane count from 1938-2009, use population growth models to project when N_p = 1000.

https://commons.wikimedia.org/wiki/File:Grus_americana_Sasata.jpg

Unit 2: Introduction to Modeling Using Difference Equations

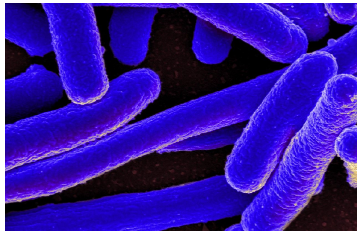
Island Biogeography



Pharmacokinetics

Invasive Species

Lab: Population Dynamics



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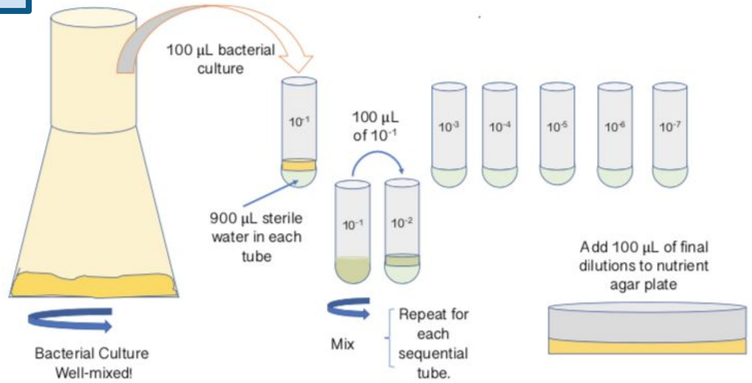
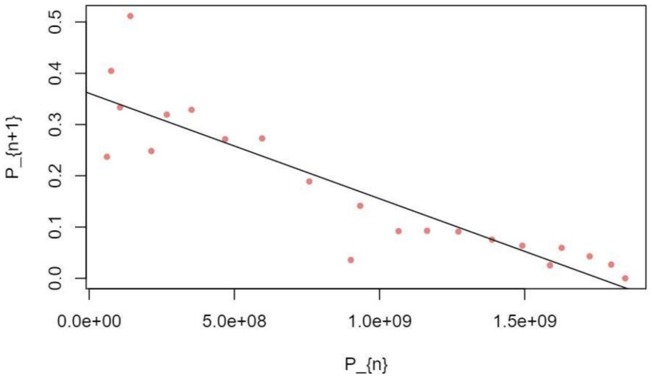
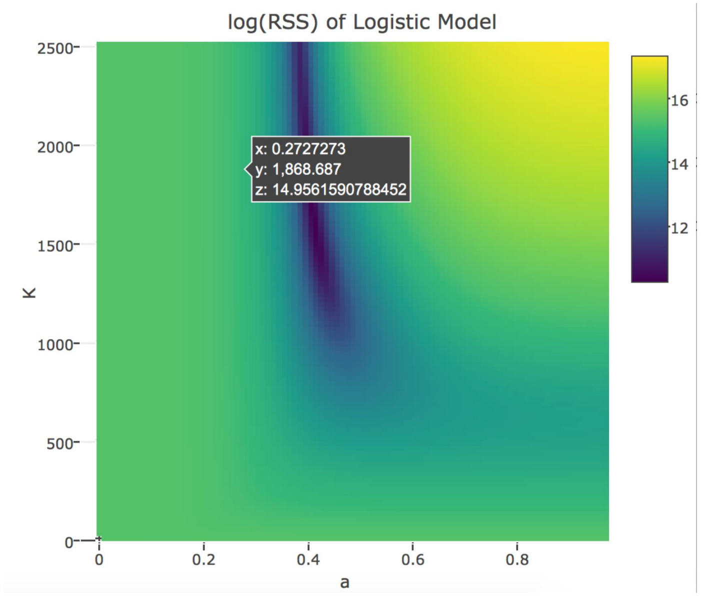




Unit 3: Differential Equations: Model Formulation, Nonlinear Regression, Model Selection
(serious modeling & lots of basics!)



Exercises:

Math & Model concepts
- rates vs rate constants
- mass action
- Feedback -- positive & negative
- Saturation

Investigating parameter space with sliders

Using sum of the squared residuals (RSS) for nonlinear fits

Heat maps

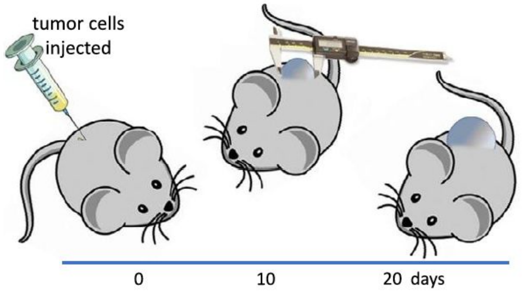
Model selection -- one tool AICc

- "filling the bathtub"
- rate constants
- Predator - prey models
- Tumor Immunotherapy, a system of three equations
- Apply RSS to bacteria assuming logistic growth model
- Compare models for temperature dependence of photosynthesis

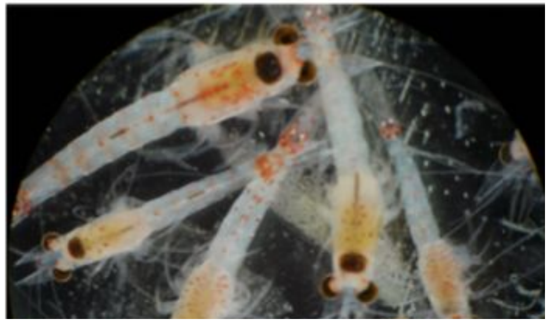
Unit 3: Differential Equations: Model Formulation, Nonlinear Regression, Model Selection

Leaf Decomposition

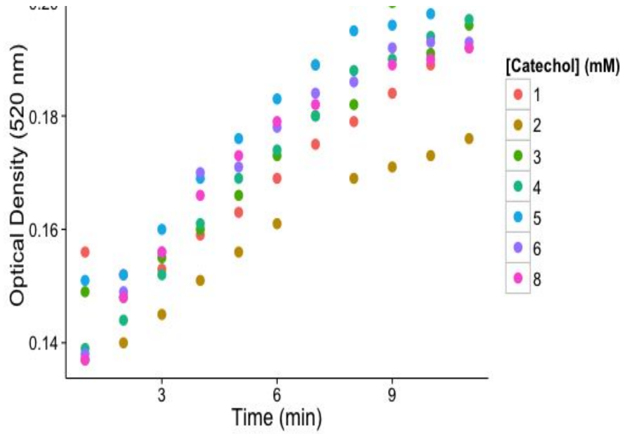
Tumor Growth Models



Predator-Prey Responses



Lab: Enzyme Kinetics



$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} ES \xrightarrow{k_2} E + P$$

$$V_0 = \frac{V_{max}[S]}{K_M + [S]}$$

Unit 4: Differential Equations: Numerical Solutions, Model Calibration, Sensitivity Analysis

Exercises:

*Numerical solutions starting with logistic growth.

*Sensitivity analysis
- Global sensitivity
- Local sensitivity

with simple logistic growth…

Spruce budworm population depends on forest and predation by birds.

Tumor growth with carrying capacity dependent on volume … two differential equations.

Putting it all together: Ebola virus infects cells

1. Frame the model
2. Numerical solution using best guess estimates
3. Data are viruses vs. time
4. Fitting parameters/calibrating
5. Sensitivity analysis

$$\frac{dU}{dt} = \lambda - \alpha U - \beta UV$$

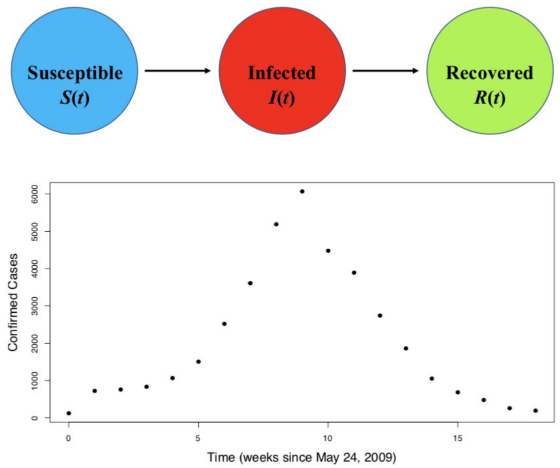
$$\frac{dI}{dt} = \beta UV - \delta I$$

$$\frac{dV}{dt} = \rho I - cV$$

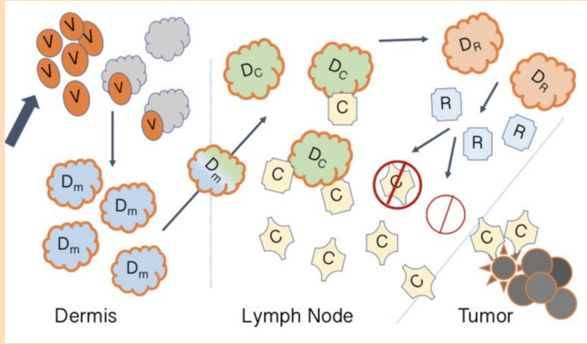
https://www.for.gov.bc.ca/RSI/ForestHealth/guide/WSBM.htm

Unit 4: Differential Equations: Numerical Solutions, Model Calibration, Sensitivity Analysis

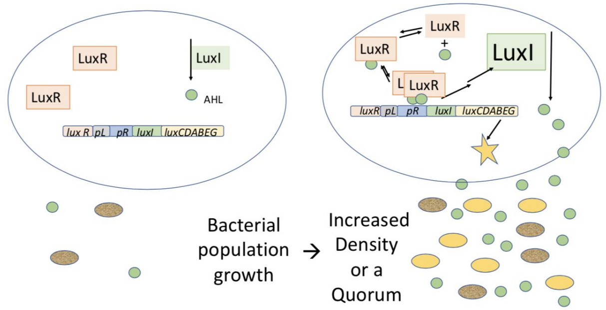
SIR: 2009 Influenza



Cancer Immunotherapy



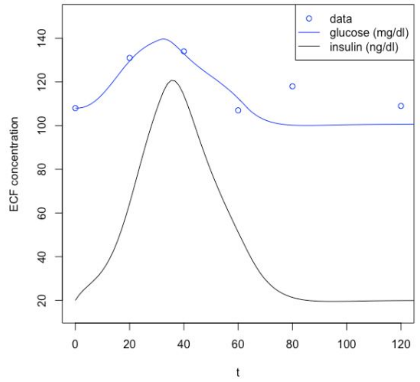
Quorum Sensing



Lab: Homeostasis: Glucose & Insulin

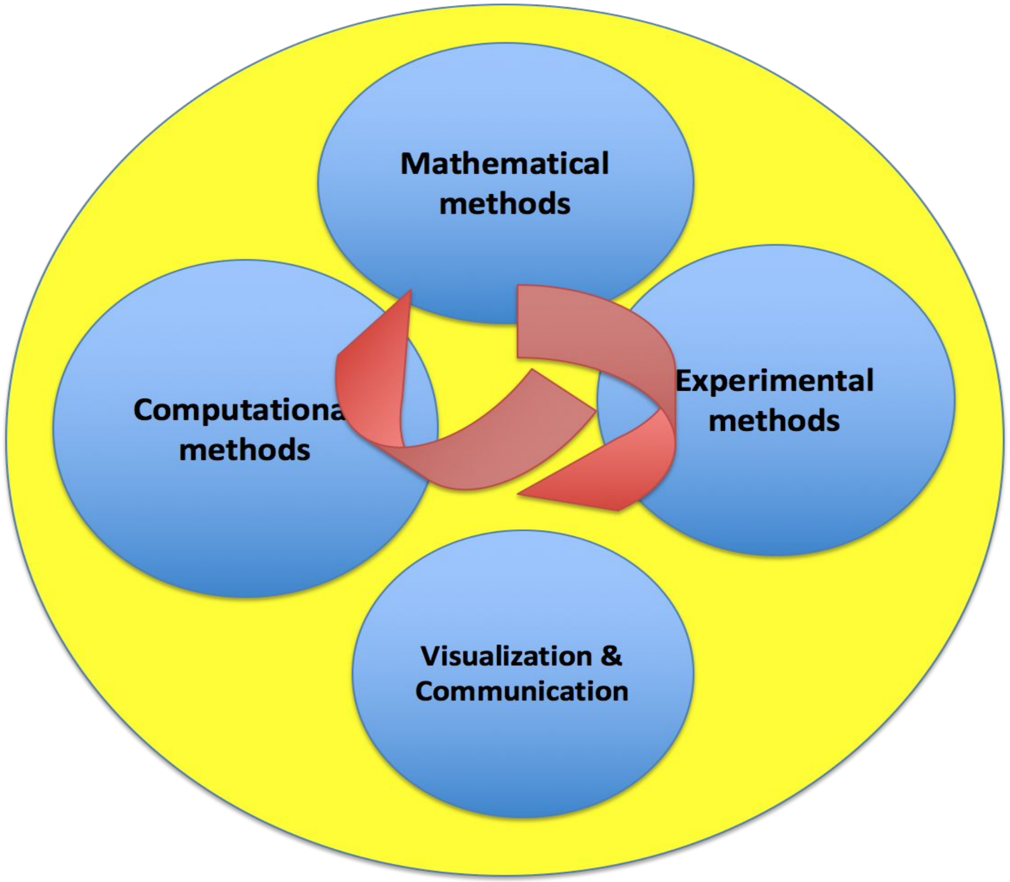
$$\frac{dG}{dt} = \text{glucose delivery} - \text{glucose uptake}$$

$$\frac{dI}{dt} = \text{insulation secretion} - \text{insulin breakdown}$$



Culminating Project:

Students generate and complete small independent projects in interdisciplinary teams



Preparation

- Paper writing exercises from data set & prompt

Steps:

- Hypothesis & research plan including data analysis
- Team oral presentation

Topics:

- Variation on Labs
- Extensions of Case Study Topics

Frequently asked questions!

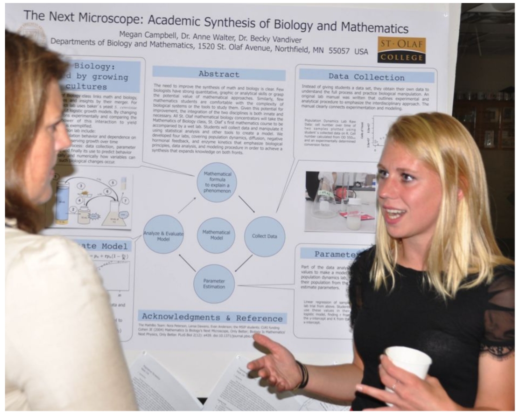
- I've never taught a lab before!
  a. Not essential but worth trying even if only one.
  b. Technical support & alternative labs including "dry" activities in the end of the book.
  c. partner with a biologist!
  d. hire a biology major as a teaching assistant

- Is team teaching necessary?
  a. No but it is fun.
  b. Problems come alive with biologist and math more nuanced with a mathematician.
  c. Individual exercises can be modified for use in many classes without a "team".

- How is team teaching supported?
  a. Limiting expectations so increase in FTE is minimal (e.g., 4 hrs/week)
  b. cross-listing, general education requirements

Acknowledgements

- Matt Richey (former St. Olaf Associate Dean)
- Nora Peterson, Megan Campbell and Lansa Dawano



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hhmi Howard Hughes Medical Institute

- Sanft, Rebecca, and Anne Walter. "Experimenting with Mathematical Biology." PRIMUS 26.1 (2016): 83-103.
- https://wp.stolaf.edu/mathbio/
- Sanft, Rebecca, and Anne Walter. *Exploring Mathematical Modeling in Biology Through Case Studies and Experimental Activities*. Academic Press, 2020.

Assessment: Quotes from Course Reflection Papers

"I am taking an introductory computer science class next semester. My decision to do so was based in part on my experiences in this class; I want to have the capability to utilize computing power to explore real-world systems."

"Next year I will be working as an ecological research assistant and I fully expect this class to influence the way I think and do work."

"The process of modeling taught me greater intentionality in my problem-solving. The math bio course has pushed me to think in ways that are not the most intuitive for me. I think I have become a stronger student because of this."

"This class gave me confidence in my ability to collaborate with students/colleagues in other fields."