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STUDENT VERSION
MODELING INTRAOCULAR GAS BUBBLES IN RETINAL SURGERY PATIENTS

Brian Winkel
Director SIMIODE
Cornwall NY USA

STATEMENT

During some vitreoretinal surgeries the opthalmological surgeon will inject a gas, sometimes air, but often various mixtures of air and sulfur hexafluoride (SF_6) or perfluorocarbons (C_nF_{2n+2}), into the fluid within the vitreous cavity of the eye (the interior of the eyeball) to help the healing process. This creates an intraocular bubble to keep pressure on the surgically treated area and provides a tamponade effect that causes functional closure of the retinal breaks, retinal detachment, macular holes, etc. See [8] for surgical procedure and details.

> Gas is now used quite frequently in many, if not most, operations; either approximately 16-20% mixture of gases (SF_6 and air or C_3F_8 and air) depending upon the case. Air, by itself, is very rarely used.[5]

By tamponade we mean the “pathologic compression of a part.”[4] In effect, ophthalmologists seek to determine “whether the intraocular bubble will tamponade the retinal break sufficiently long for a chorioretinal adhesion to form around the break”[9].

We quote from the medical literature concerning the need to monitor the size of the bubble.

> “Intravitreal gas is commonly used in conjunction with vitrectomy and retinal reattachment to tamponade retinal breaks in the treatment of complicated retinal detachments. For a given retinal break, the size and duration of the intravitreal gas bubble determine whether effective tamponade of the retinal break is present sufficiently long enough to maintain apposition of the retina and retinal pigment epithelium. It becomes important to be able to accurately predict the kinetics of disappearance of intraocular gas bubbles to make an appropriate decision regarding the amount and type of gas to be used.”[12, p. 609]

Moreover, surgeons need to know what gas to use and how much gas to use.

> “Predictability of the kinetics of intraocular gases is important since the surgeon must choose the type of gas and the size of the intraocular gas bubble to achieve adequate duration of intraocular tamponade of the retinal break.”[9, p. 691]

The general shape of the eye is presumed to be a sphere of radius 1 cm and the gas bubble is known to have the following shape: a flat circular disk on the bottom (meniscus) with a domed top in contact with the eyeball. The bubble floats at the top of the eyeball and when the patient is standing upright this bubble is bounded below by the flat disk meniscus, parallel to the ground. However, due to the optics of the eye, the bubble appears to the patient to be on the bottom of the eyeball with a flat top. The patient can see the exact shape of the bubble and its progress throughout the recovery period after surgery.

Doctors monitor the size of the bubble in order to determine if the bubble is suitable to provide the tamponade effect. In Figure 1 we see one approach to measuring the size of the bubble by recording the height of the meniscus, i.e. the flat base of the bubble from the base of the eyeball, and then computing the height at time t (in days) as a percentage of the vertical diameter of the eye. This is easily converted to centimeters using the assumption that the typical eyeball is 2 cm in diameter and so $h = h(t)$, the actual height of the meniscus in cm, is simply one minus the percentage observed times 2 cm, i.e.

$$h = \left(1 - \frac{\text{Percent}}{100}\right) * 2\,. \tag{1}$$

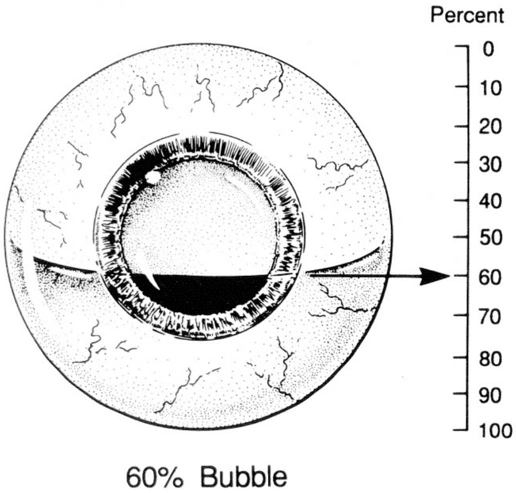

Figure 1. “The meniscus height of an intraocular gas bubble is estimated clinically as a percentage of the vertical diameter of the eye seen through the dilated pupil with the plane of the cornea perpendicular to the ground.”[12, p.609]

Usually, the surgeon injects sufficient gas to occupy all of the spherical vitreous volume of the eyeball. Thus we shall assume the bubble occupies the entire volume of the eyeball (vitreous cavity) initially, i.e. $h(0) = 0$.

Over a period of time the gas diffuses out of the eyeball and the full vitreous fluid level returns to the patient's vitreous cavity. A reasonable amount of time for such a bubble to go to a negligible level (say less than 0.1% of the volume of the eyeball) is 60 days. Interestingly, in patients with diabetes or other reasons that do not allow the intraocular fluid to exit normally, the bubble lasts much longer.[5]

Patient Data

In personal correspondence with Dr. John T. Thompson [10] we received a set of human data which was used in the study found in [9]. We have placed the complete data set in a supplemental file to this paper on the SIMIODE website under the name 1-30-EyeData.xls [11]. This file contains an explanation of the exact nature of the data file, as well as the data itself. We selected several patient data sets for our work here and offer them in Table 1 (see end of this article).

Initial Model and Rationale

The medical literature suggests that the volume of the intraocular gas bubble in the vitreous cavity, $V = V(t)$ in cm^3 at time t in days can be modeled by the following exponential decay model

$$\frac{dV}{dt} = -kV \,, \qquad V(0) = \frac{4}{3}\pi(1^3) \,. \tag{2}$$

NB: The radius of the typical eye is presumed to be 1 cm.

Indeed, in calculating the absorption of the bubble there are two models studied: "(1) natural logarithm of bubble volume vs. time, and (2) surface area of contact of the intraocular bubble vs. time. . . . With model 1, the absorption of nonexpansile intraocular gas bubbles has been reported to approximate a first-order exponential equation with respect to intraocular gas bubble volume vs time [12, 7, 1]."[9] From a summary plot the author observes "Compared with model 2, model 1 shows a more rapid initial decline in absolute volume but a slower decline in absolute volume when the intraocular bubble is small."[9, p. 689] There is support for the exponential decay model in practice through these observations due to its ease of use and the use of the common notion of half-life for the height of the bubble.

We consider data from several patients, identifying them by patient number as found in [10] for analysis. In Table 1 we offer the complete data set for these patients from [10].

In Figure 2 we plot the meniscus height of the intraocular gas bubble for Patient 126 as estimated clinically as a percentage of the vertical diameter of the eye seen through the dilated pupil with the plane of the cornea perpendicular to the ground. We use (1) to convert from percent of maximum height of 2 cm to actual height in cm.

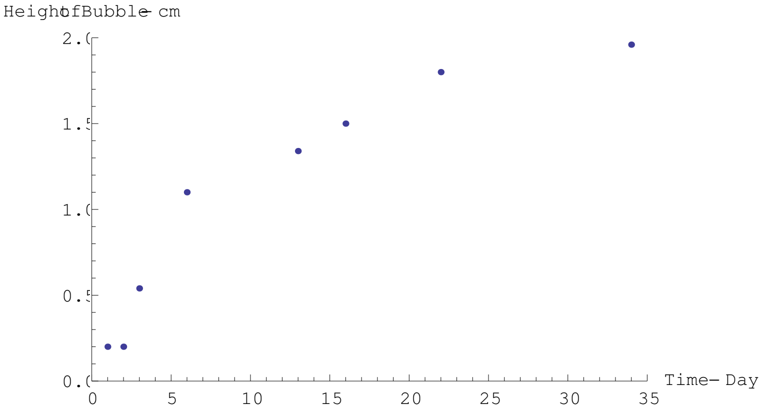

Figure 2. Plot of the meniscus height of the intraocular gas bubble for Patient 126.

Figure 3 shows the computed volume of the bubble for the observed data for Patient 126 as a function of time in days and would appear to motivate the immediate conjecture of an exponential decay model, i.e. the rate of change in volume per unit time in days is proportional to the volume itself.

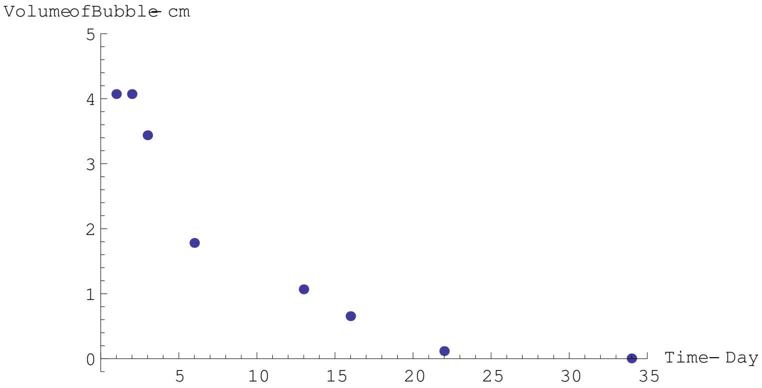

Figure 3. Plot of the volume of the intraocular gas bubble for Patient 126.

Activity 0

Explain formula (1) and then show how one obtains the volume of gas of the bubble depicted in Figure 1 given its height.

Now, in Activities (1) - (5) below perform your analyses using Patient 126 data and then take one other patient from the file 1-30-EyeData.xls [11] and perform the same analyses.

Activity 1

Either by examining some of the data in 1-30-EyeData.xls [11] or reasoning and some assumptions (state them) defend why (2) is a reasonable model.

Activity 2

Pick several patient's data from the file 1-30-EyeData.xls [11] and model the data with (2). Determine the rate constant k for several patients and compare them. Indeed, the decay rate $k > 0$ is determined for patient data under various lens scenarios and the resulting model, so pick identical and different scenarios or conditions for the patient.

Activity 3

Think hard about this physiological situation. The gas in the center of the bubble goes nowhere, but the gas along the wall of the vitreous region in the eyeball could escape. With that in mind offer up a different model than (2). Here too, pick several patient's data from the file 1-30-EyeData.xls [11] and model the data with (2). Determine the parameters in your model for several patients and compare them. Indeed, the parameters may be determined for patient data under various scenarios and the resulting model, so pick identical and different scenarios or conditions for the patient.

Activity 4

Let us refine our model in Activity 1-3 by considering ALL the different ways the gas can escape the vitreous region of the eyeball and reflect that in a final revised model. With this new model in mind offer up a different model. Here too, pick several patient's data from the file EyeData.xls [11] and model the data with this newest model. Determine the parameters in your model for several patients and compare them. Indeed, the parameters may be determined for patient data under various scenarios and the resulting model, so pick identical and different scenarios or conditions for the patient.

Activity 5

Compare results from all your models and attempt to defend why the opthalmological profession uses the exponential decay model with half-life information in its patient analyses.

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126	I	C	0.1	6	25	1	0.90	4.5850	1.5228	0.2
126	I	C	0.1	6	25	2	0.90	4.5850	1.5228	0.2
126	I	C	0.1	6	25	3	0.73	3.8377	1.3449	0.54
126	I	C	0.1	6	25	6	0.45	2.0266	0.7064	1.1
126	I	C	0.1	6	25	13	0.33	1.2460	0.2199	1.34
126	I	C	0.1	6	25	16	0.25	0.7830	-0.2446	1.5
126	I	C	0.1	6	25	22	0.10	0.1460	-1.9241	1.8
126	I	C	0.1	6	25	34	0.02	0.0062	-5.0832	1.96
179	P	C	0.2	12	2	1	0.85	4.8872	1.5866	0.3
179	P	C	0.2	12	2	2	0.80	4.6630	1.5397	0.4
179	P	C	0.2	12	2	3	0.75	4.3912	1.4796	0.5
179	P	C	0.2	12	2	6	0.70	4.0791	1.4059	0.6
179	P	C	0.2	12	2	10	0.70	4.0791	1.4059	0.6
179	P	C	0.2	12	2	20	0.45	2.2137	0.7947	1.1
179	P	C	0.2	12	2	36	0.28	0.9961	-0.0039	1.44
179	P	C	0.2	12	2	42	0.28	0.9961	-0.0039	1.44
186	A	C	0.2	13	4	1	0.75	3.2540	1.1799	0.5
186	A	C	0.2	13	4	2	0.75	3.2540	1.1799	0.5
186	A	C	0.2	13	4	3	0.70	3.0223	1.1060	0.6
186	A	C	0.2	13	4	6	0.65	2.7730	1.0199	0.7
186	A	C	0.2	13	4	13	0.55	2.2338	0.8037	0.9

Table 1. Observational data for several patients referred to by patient number in which we see, for example, Patient 126 has an Intraocular lens (I); gas used for the bubble was Perfluoropropane gas (C_3F_8) (C); concentration of the gas was 10% (0.1); patient was assigned series number (6); patient was assigned a number in that series for the study (25); times of observation in days were 1, 2, 3, 6, 13, 16, 22, and 34; the percentage (decimal) of meniscus height of the bubble (beginning with 0.90); the computed bubble volume (cm^3) from meniscus height (beginning with 4.5850); natural logarithm of bubble volume (beginning with 1.5228); and actual height of the bubble in cm (beginning with 0.2) (added by this author). Patients 179 and 186 have other conditions denoted by P and A respectively. [11].