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STUDENT VERSION
Spring Inverse Problem

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STATEMENT

This is an example of an *inverse problem* in which we are given data and we seek to estimate a parameter in the model so as to fit this data with a model and thus validate the model.

Suppose we have observations (Table 1) on position, $y(t)$, in cm, vs. time t in s of a small oscillating spring in a dashpot. We know the mass is $m = 1$ g and the spring constant is $k = 20$ dyne/cm. We can measure those.

t s	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y(t)$ cm	2.16	1.77	1.40	1.08	0.71	0.39	0.18	0.03	-0.06	- 0.01	-0.10

Table 1. Data on the displacement of a mass on the end of a spring.

$$m \cdot y''(t) + c \cdot y'(t) + k \cdot y(t) = 0\,, \quad y(0) = 2\,, \quad y'(0) = 0\,. \tag{1}$$

The differential equation model in (1) discribes the motion of the mass at the end of the spring. We displace the spring 2 cm from its static equilibrium and give it an initial velocity of $y'(0) = 0$, i.e. we just release it. We seek to find the resistance coefficient c in dyne/(m/s) where these parameters are in (1)

Discuss how you would determine c. Offer up several ideas. Carry out one approach at least. Confirm your estimate for c in some manner. If you have offered up several approaches compare your results. How does your model with your c value compare to the data?