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STUDENT VERSION

Thanos Population Dynamics

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Abstract: In the end of the "Avengers Infinity War," the villain Thanos snaps his fingers and turns half of all living creatures to dust with the hope of restoring balance to the natural world [1]. How does this affect the long term behavior of various species? Investigate the validity of his claim by modeling various population dynamics such as unconstrained and constrained growth.

Keywords: Population dynamics, equilibrium values, stability

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STATEMENT

In the 2018 blockbuster, Marvel Studios' "Avengers: Infinity War," the villain Thanos snaps his fingers and turns half of all living creatures in the universe to dust [1]. He was concerned that overpopulation on a planet would eventually lead to the suffering and extinction of the entire population. This is evident in the following quote from Thanos.

"Little one, it's a simple calculus. This universe is finite, its resources finite.

If life is left unchecked, life will cease to exist. It needs correction."

1. There is a bit to unpack in Thanos's quote. What are some of the assumptions that Thanos is making?

In this activity, we will investigate the validity of Thanos's claims using mathematical models for population dynamics.

Model 1

First, we will consider the following initial value problem

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP, \quad P(0) = P_0,$$

where P is the population at time t, and k and P_0 are constant parameters.

2. Interpret the meaning of this differential equation.

3. Solve the initial value problem and determine what would happen to a population in the long run. Explain why your answer makes sense in terms of the differential equation.

4. This model is called **unconstrained growth**, since the population grows without bound. Under what assumptions would it be appropriate to use this type of model? Does this model the situation Thanos is describing?

5. Thanos's plan is to eliminate half of all living creatures in the universe. What would happen if the population size was suddenly cut in half? How could that be represented with this model? What parameters would change?

Model 2

Next, consider the following initial value problem

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP\left(1 - \frac{P}{L}\right), \quad P(0) = P_0,$$

where P is the population at time t, and k, L, and P_0 are positive constant parameters.

6. How does each parameter affect the growth of the population?

7. For what value(s) of P, if any, would the population stay constant? This value will be called an **equilibrium solution**.

8. Note that an equilibrium solution is considered **stable** if all solutions close to the equilibrium value approach the equilibrium. Otherwise, the equilibrium value is **unstable**. For each equilibrium value, determine the stability.

9. Solve the initial value problem and determine what would happen to a population in the long run. Explain why your answer makes sense in terms of the differential equation.

10. Thanos's plan is to eliminate half of all living creatures in the universe. How would halving the initial population impact the overall dynamics of the system?

11. Model 2 is called **constrained growth**, since the population grows or decays until it reaches a carrying capacity. Under what assumptions would it be appropriate to use this model? Do these assumptions seem more or less reasonable than the first model for describing Thanos's version of reality?

Homework

Thanos claims to be a logical person. In the sequel, "Avengers: End Game," time travel is used to undo Thanos's work. Suppose you go back in time and work your way through to become a part of Thanos's inner circle. Prepare a report to Thanos to encourage him to rethink his plan. Your report should discuss the assumption and results from both of the mathematical models discussed.

REFERENCES

[1] *Avengers: Infinity War*. Marvel, 2018.