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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

STUDENT VERSION
SALT COMPARTMENTS

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STATEMENT

Suppose we have two tanks of liquid, Tank 1, holding 100 liters of pure water, and Tank 2, holding 200 liters of pure water - reasonable size fish tanks you might have in your home/office. We dump 100 g of salt (NaCl) into Tank 2, thoroughly dissolve and mix so the salt is uniformly distributed in the tank. Next we start a pumping action in which we pump 8 l/min of thoroughly mixed saline solution from Tank 1 to Tank 2 and at the same time we pump 8 l/min of thoroughly mixed saline solution from Tank 2 to Tank 1, thus keeping volumes of water in the respective tanks the same throughout our experiment. Let us assume that a stirrer in each tank keeps the solution throughly mixed so that at the very instant saline solution enters a tank its salt is thoroughly mixed in that tank.

Let $x(t)$ be the amount of salt in g in Tank 1 at time t in min and $y(t)$ be the amount of salt in g in Tank 2 at time t in min.

a) Set up a system of linear differential equations with initial conditions to describe the flow of salt in each tank at time t min.

b1) Solve the system using an eigenvalue/eigenvector approach and plot both $x(t)$ and $y(t)$ over a 60 minute time interval.

b2) Solve the system by eliminating one of the variables $x(t)$ or $y(t)$ and solving for the remaining variable. Then use that solution to solve for the other variable. Plot both $x(t)$ and $y(t)$ over a 60 minute time interval.

c) From your plot describe what is happening with the salt in solution and tell why you could have predicted this at the start without any differential equations!

d) How can the eigenvalues and eigenvectors tell you the observed phenomena in (c) would happen?

e) Use *Mathematica*'s `DSolve` to solve these equations and pick out the eigen information in your solutions. Also plot these solutions showing that your results are identical to those in (b1) and (b2).

f) Compute the total amount of salt in your solution from your solutions at all time. Why is this to be expected?