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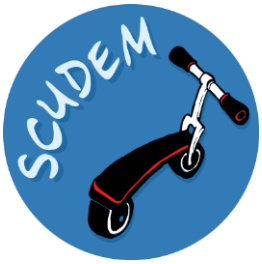
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

SCUDEM – SIMIODE Challenge Using Differential Equation Modeling Problems and Additional Issue for

SCUDEM I (October 2017), SCUDEM II (April 2018), and SCUDEM III (October 2018)

See SCUDEM home page for details of SCUDEM IV 2019, 9 November 2019.



General Information

For each SCUDEM event three student teams from high school, two year colleges, and four year undergraduate colleges and universities, select one of three problems to work on for 8 days at their home competition and then assemble as local site nearby for engaging day of sharing their results, enjoying camaraderie with other students and faculty coaches, working on an additional issue for the model of their choice, participating in a fun MathBowl, and getting immediate feedback and awards for their efforts.

Upon arrival at the local site teams submit a two page executive summary, receive an Additional Problem with the instructions, "You are not to redo your model, but just indicate how the additional issue for your model might be incorporated in your mathematical formulation and the results you might expect." They incorporate their responses to the additional issue in their previously prepares ten minute presentation for judging.

The three problems offered at each SCUDEM are in the areas of (a) physics/engineering, (b) life science/chemistry, and (c) social science.

Complete details on all aspects of SCUDEM including statements of the problems, commentaries by the author of the problems, and all student submissions are offered in the respective SCUDEM event sections at the SCUDEM home page.

In the following pages we offer the statements of the Problems and the Additional Issues for each SCUDEM event for all three problems, A, B, and C.

SCUDEM I 2017 - Problem A - Going Viral

Advertisers strive to reach the greatest number of people using the least amount of resources. The prevalence of social media and easy access to digital media has resulted in the development of strategies to create advertisements that go "viral." Advertisers want a large number of people to share their message in ways that the number of people seeing the message grows rapidly in a short time.

This has been done in a variety of ways. One way is to target specific groups of people who are known to have broad audiences that can spread a message quickly. Another way is to make an advertisement available to a great number of social media outlets in a short time span. The hope is that the advertisement will be shared by enough people that the sharing itself will gain momentum and increase rapidly.

A good deal of research has been conducted to model the way information spreads. Many of these models focus on the structure and size of the network of people. In practice the networks of people are already established and can be vast and quite complicated. Is it possible to construct a simpler model that can describe the way the number of people are exposed to an idea and predict viral growth?

In particular, an advertising company would like to know the minimal amount of resources that can be spent and result in a rapid expansion of the number of people exposed to a message. For example, is there some minimal number of people who must be exposed to their message in a short time to cause the message to go viral? If an advertisement does not go viral quickly after release how likely is it that it will never go viral, or is it possible to add resources in a way that can promote an advertisement so that it does go viral if at first it does not appear to be successful?

Four additional issues to be incorporated in your model are given below. Choose one issue from the list and explore how your model can be adapted to include the new aspect.

1) One aspect of the campaigns that are designed to rapidly disseminate a piece of information is how to detect when it is happening. Given your model, is it possible to detect when an agency is attempting to manipulate the system as a way to force the rapid spread of some information? What kind of monitoring should be put in place to detect this kind of manipulation predicted in your model?

2) The phenomena of viral campaigns has been identified and multiple groups are trying to exploit ways to rapidly share information. Given that there are many groups that are trying to use these methods, can your model be modified to examine what happens when two competing ideas are being manipulated? For example, suppose that two agencies are trying to generate excitement about two similar products or events. How can you adapt your model and what will it predict with respect to two competing agencies?

3) The phenomena of rapid dissemination became more prevalent as new communications platforms were implemented. For example, Twitter made it possible for small snippets of information to be shared rapidly across a large network of people in a short time. What will the impacts of various changes to these platforms be to your model? For example, if a social network implements new methods to curb some accounts and impacts the methods you propose, how will it change the results that follow for people who implement your recommendations? What would happen if a new social platform becomes more widely used that is faster and draws people away from other platforms? Will it change your recommendations?

4) An organization is considering using your model but wants a change. They want to also have a prediction as to how long it will take for the information they want to spread to hit the peak transfer rate. They want the model adapted so that if the methods in use will not result in a quick dissemination they can make changes to the approach to insure that the information is spread more rapidly. Determine what should be monitored, and how that additional information can be used to make changes to the recommendations.

SCUDEM I 2017 - Additional Issue for Problem A - Going Viral

An organization is considering using your model but wants a change. They want to also have a prediction as to how long it will take for the information they want to spread to hit the peak transfer rate. They want the model adapted so that if the methods in use will not result in a quick dissemination they can make changes to the approach to insure that the information is spread more rapidly. Determine what should be monitored, and how that additional information can be used to make changes to the recommendations.

SCUDEM I 2017 - Problem B - Drug Interactions

Patients undergoing medical treatment may be given a combination of drug therapies, and the information about the interaction between drugs can be limited[1]. One commonly prescribed class of drugs is diuretics which promote kidney function as a way of removing water and sodium from the circulatory system. The way that a diuretic can interact with other drugs can be quite complicated and depends on the type of diuretic.

We ask that you examine the simpler issues of the reduction or removal of a compound from within a patient's system. A drug can be reduced within a patient's system in several different ways. For example, it can be metabolized and broken down within the liver, and a patient's kidneys can directly remove it from the bloodstream.

You are asked to provide an analysis that will give direct guidance for the administration of a drug. The patient will be given a diuretic to relieve symptoms associated with heart disease. At the same time a patient will be given another drug to help treat other symptoms. The goal is to maintain a consistent level of both drugs within a patient's circulatory system.

The issue is that the diuretic will promote a more rapid removal of both drugs from the patient's system, but at the same time the other drug must be maintained at levels that are effective and safe. The medical staff would like to know how to balance the administration of the two drugs. They need to know what schedule and what dosages are appropriate for a given situation.

Reference

Egger, S. S., J. Drewe, and R. G. Schlienger. 2003. *Eur. J. Clin. Pharmacol*. 58: 773-778. doi:10.1007/s00228-002-0557-z.

SCUDEM I 2017 - Additional Issue for Problem B - Drug Interactions

As part of the treatment a patient may be given additional fluids to counteract the loss of fluids due to the diuretic. Some patients will be given the additional fluids using an intravenous fluid therapy, and the fluids will be introduced at a relatively constant rate. Other patients will be asked to drink large amounts of water at regular time intervals. Adapt your model to be able to predict what will happen when the concentrations of the drugs can change due to different rates of water intake.

SCUDEM I 2017 - Problem C - Game Play

A new game is proposed for a hand-held device. The game needs to be relatively simple and the computational resources required must be kept small. The proposed game is a network based game, so two players can play at the same time on their separate devices.

The concept is that there is an animated ping pong ball that moves through an obstacle course and its motion is controlled in turn by the two players. The first player moves a paddle to hit the ball into the obstacle courses. When the ball reaches the other end of the obstacle course the second player must move a paddle to return the ball to the first player who then maneuvers the ping pong ball back to the second player on the other end of the obstacle course. The process repeats until a player is unable to return the ball, thus giving the player who misses one point. Much like table tennis when a player reaches 21 points and the margin in points is 2 or more the player with the higher points loses and a winner is declared. Service alternates and either player can score on service.

The ping pong ball must move in two dimensions, left- right and forward- backward. The ball is exposed to friction so it will slow down if no other forces act on it. The players can provide a boost by pushing the ball with a force that can be forward or backward and a force that can be left or right. The player must use the forces to move the ball through the obstacle course as quickly as possible while avoiding any obstacles.

You are asked to develop the system of equations that describe the motion of the ping pong ball. You should also provide recommendations to the software developers. They need to know the levels of force necessary to make the game interesting, yet still playable. They also need to know what limitations should be put in place so that the game will be challenging, but not too easy. For example, there should be a limit on the amount of propellant available to apply force to the ping pong ball. The developers want players to develop a strategy on how to use the limited propellant allowed. Since these decisions depend on the number and size of the obstacles you should also provide recommendations about the obstacle course and overall game play that will make the experience as enjoyable as possible.

SCUDEM I 2017 - Additional Issue for Problem C - Game Play

Provide an analysis of which of the possible factors of the game result in the biggest changes for small changes. For example, does a small change in the number of obstacles have a large impact on the game? Does a small change to the amount of propellant make a big difference? Use your analysis to augment your report to indicate which factors will require the most testing and review.

SCUDEM II 2018 - Problem A - Sorting Recyclables

When recycling began in the United States many places required that people separate the different kinds of materials they recycled, and the different materials were collected separately. The initial recycling rates were low, partly due to the inconvenience of having to separate and keep the materials apart. Since that time single stream recycling has become more common, and people can simply place all of their recyclable materials in a single bin. The resulting materials are then sorted at a recycling facility.

Recyclable materials generally go through a number of stages to separate the different materials. One stage is used for the materials that consist of either paper or cardboard materials. These are difficult materials to separate, and a good deal of this material is sorted by hand. The question to explore is whether or not a simple process can be developed that will help separate a large percentage of the materials, specifically paper and cardboard materials.

A simple device will be tested in which the materials will be dropped from a great height, and a fan will blow air across the stream of falling material. Determine the minimal height and wind speed that can be used to separate 30%-40% of the paper that is in the falling column of material. For our purposes you should assume that the distribution of the paper and cardboard items are relatively uniform but make sure your assumptions are explicitly stated. The goal is to establish the feasibility of the general idea before proceeding to a more complex situation.

SCUDEM II 2018 - Additional Issue for Problem A - Sorting Recyclables

Which aspect of your model results in the largest difference in sorting quality if that aspect undergoes a small change. For example, does a small change in the height of the drop make a bigger difference in the final results as compared to a small change in the wind speed associated with the fans?

SCUDEM II 2018 - Problem B – Alarm Bells

Prey animals have to strike a balance when deciding whether or not to flee a potential predator. Moving away in a hurry can expend a great deal of energy, and some prey animals only have a limited ability to detect a larger animal's intentions. As an example, this dilemma was explored in a recent paper [1]. The researchers in this particular study examined the response of larval zebra fish and found that both the size of the potential predator and the rate the size changed influenced how the larval fish responded to a potential threat.

We ask that you explore the general phenomenon and develop a system of ordinary differential equations that mimics this behavior. The basic idea is that relatively simple organisms must make complex decisions and do so with the least possible resources. Is it possible for an organism to
incorporate a relatively small amount of information, such as the size and he rate of change of the size of a potential threat, and then make this decision based on a simple model of ordinary differential equations? If so, what does your model imply about repeated exposures? Does the frequency of those exposures in a short time have an impact on prey response?

A good starting point for understanding the basic ideas behind these models can be found in a paper by Tyson [2]. The models in this paper demonstrate how a response can be determined from a single input. The question we ask is, "How can two or more inputs be incorporated together to enable a simple organism to decide whether or not to flee?"

References

1. Bhattaacharyya, K., D. L. McLean, and M. A. MacIver. 2017. Visual Threat Assessment and Reticulospinal Encoding of Calibrated Responses in Larval Zebrafish. *Current Biology*. 27(18): 2751-2762. http://dx.doi.org/10.1016/j.cub.2017.08.012. Also see https://www.eurekalert.org/pub_releases/2017-09/nu-net090617.php .

2. Tyson, J. J., K. C. Chen, and B. Novak. 2003. Sniffers, buzzers, toggles and blinkers: dynamics of regulatory and signaling pathways in the cell. *Current Opinon in Cell Biology*. 15(2): 221-231. https://bioinformatics.cs.vt.edu/~murali/teaching/2003-fall-cs6104/papers/modelling/tyson-current-opinion-cellbio-vol15-2003-sniffers-buzzers-toggles-blinkers.pdf .

SCUDEM II 2018 - Additional Issue for Problem B - Alarm Bells

Determine the best strategies that a predator can use to successfully catch a prey animal that uses your model to determine when to flee.

SCUDEM II 2018 - Problem C - Modeling the Cool Kids

One common stereotype about interactions of students at high school and college is that students self-organize into different social groups or cliques. The way different groups of people come together and form social bonds is a well-documented phenomena and not just an old Hollywood trope. One aspect of this phenomenon that is not well understood is how the groups change over time. As an example, a group of researchers recently found that solely examining academic performance was a good predictor of how close people could be tied together [1]. They also found that the resulting networks that bind people can reorganize in time, based on a small number of factors.

The question to explore is how can the dynamics of social interactions and grouping be modeled and examined in time. Can an ODE model that only makes use of a very small number of social factors mimic complex group dynamics that change in time? What does your model predict for how quickly groups can form and change? What will happen in the long run? Finally, what happens as the number of groups increases? For example, will just having two groups be more stable than having three or more groups that interact?

Reference

Smirnov, Ivan and Stefan Thurner. 2017. Formation of homophily in academic performance: Students change their friends rather than performance. Published: August 30, 2017. https://doi.org/10.1371/journal.pone.0183473 .

SCUDEM II 2018 - Additional Issue for Problem C – CoolKids

How can your model be extended or changed to examine the dynamics of how people identify with political parties? (Your model will have to include the option to not be part of any group or party.) How do the dynamics change in this new situation and what changes are necessary to demonstrate cross-over voting?

SCUDEM III 2018 - Problem A - Conflict Between Patrilineal Clans

Roughly 7,000 years ago the genetic record for humans indicates that there was a dramatic decrease in the variation in Y chromosomes [2]. It appears that the number of people carrying the Y chromosome decreased to one twentieth of their previous number. There is not an indication for a corresponding drop in the number of people who do not carry the Y chromosomes.

A group of undergraduate students at Stanford University recently [1,2] hypothesized that the reason for the sudden decrease in males is due to the development of patrilineal clans among humans, and the resulting strife and wars between the clans impacted males at a much higher rate than females. The group developed a mathematical model describing the genetic interactions that result from the hypothesized situation and showed that the resulting genetic trends are consistent with the current genetic variations seen in both male and female populations.

Assuming that the hypothesis is correct, develop a mathematical model describing conflict between neighboring patrilineal clans that can be used to predict the resulting population dynamics including the distribution of males and females in the human population. Based on your models under what conditions are conflicts most intense? Was the decline of patrilineal clans inevitable or is it possible to reach an equilibrium under such a social tradition?

One model for a group of two sets of males is given in the original paper [2]. The model assumes uniform mixing of the different groups of males and only one group of females. Your model should not assume a uniform pool of females, but include different groups of females associated with the clans. Additional discussion on how to extend the model to more than two groups is expected.

References

[1] Collins, Nathan. 2018. Wars and clan structure may explain a strange biological event 7,000 years ago, Stanford researchers find. Stanford University News Service. 30 May. **https://news.stanford.edu/press-releases/2018/05/30/war-clan-structubiological-event/**. Accessed 3 September 2018.

[2] Zeng, Tian Chen, Alan J. Aw, and Marcus W. Feldman. 2018. Cultural hitchhiking and competition between patrilineal kin groups explain the post-Neolithic Y-chromosome bottleneck. *Nature Communications*. Volume 9, Article number: 2077. https://www.nature.com/articles/s41467-018-04375-6. Accessed 3 September 2018. Freely downloadable. This article is licensed under a Creative Commons Attribution 4.0 International License.

SCUDEM III 2018 - Additional Issue - Problem A - Conflict Between Patrilineal Clans

What is the role of human mobility in your model? For example, how would the model change if horses were domesticated earlier? Does your model make different predictions for highly mobile groups of people, such as some Pacific Islander groups versus groups whose travels were more limited?

SCUDEM III 2018 - Problem B - Swing and a Hit

A device (see the full frontal rendering in Illustration 1) is to be constructed that will be used to permit a metal ball to roll down a ramp as one part of a larger museum exhibit for children. This ball will strike another ball attached to the end of a hanging rod which will cause the device to rotate 180 degrees and lightly strike a domino after the half rotation about the central pole. The device will be built from a vertical, one meter, central pole, and while another rigid half meter pole will extend horizontally from the top of the vertical central pole. At the end of the horizontal pole another rigid rod with a ball at the end of the rod will hang down and will be free to swing in any direction. The hanging rod will have a length of 0.80 meters.

The device will be constructed so that when it is at rest the hanging rod will hang down near the lip of the ramp as show in Illustration 1 below. Separate from the tower, a metal ball whose mass is 600g will roll down a ramp and strike another metal ball attached to the end of the hanging rod. The contraption should initially be in the configuration as shown in Illustration 1, and the moving rolling mass should be moving straight out of the page on the left lower side. The whole contraption will be free to rotate around the central, vertical pole, and it should rotate 180 degrees in order to strike the domino. With respect to the illustration it should rotate out of the page. The ball to be attached to the end of the hanging rod should just lightly strike the top of a domino that is standing up and is 0.3 meters above the floor as shown in the lower right side of Illustration 1.

Your team has been asked to determine how best to accomplish the goal. In particular, to start building the contraption the technicians will need to know the mass of the metal ball to be attached to the bottom of the hanging rod. They will also need to know where to locate the domino. Finally, they need to know at what velocity the original metal ball should be moving when it strikes the ball attached to the hanging rod in order to accomplish the desired result. The primary design constraint is that the domino should be 0.30 meters above the floor. However, if your team decides that the geometry should change then you should provide a clear case for any new recommendations.

The director of the exhibit has indicated that she would like the motion to be as dramatic as possible. In this case that means as wide a swing as possible, but also slow enough so that children can watch in anticipation. At the same time, the motion must be reproducible and reliable.

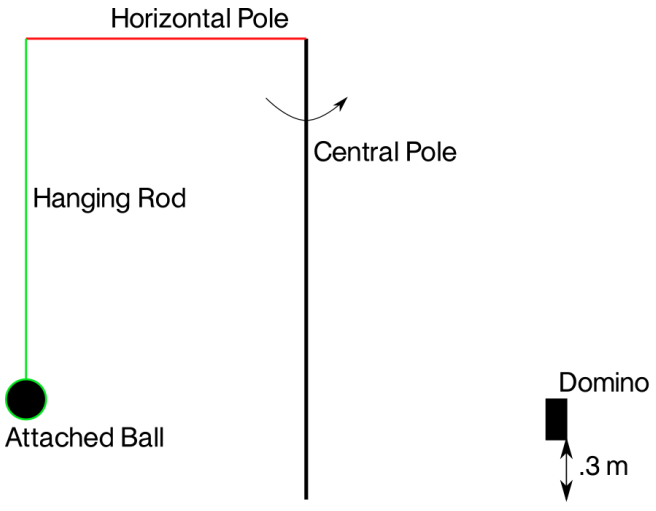


Illustration 1: Side view of the contraption.

SCUDEM III 2018 - Additional Issue - Problem B - Swing and a Hit

If you could make one design change for the device what would it be? Explain your choice and why you believe it is the best candidate for a change. What changes would you expect to see in the motion of the new device?

SCUDEM III 2018 - Problem C - Snakes in the Long Run

The sex of some reptiles depends on a number of factors including the incubation temperatures of the eggs. The dependence of temperature for Pine Snakes was examined in a paper by Burger and Zappalorti [1]. They found a linear dependence between incubation temperature (in Celsius) and the sex ratio (male/female) for Pine Snakes,

$$SexRatio \sim 0.068\,(Temp) - 0.95.$$

One question that arises is what might happen if there are rapid changes in environmental temperatures. For example, one prediction about climate change is that there will be larger variations in yearly temperatures.

Assuming this is the case, explore the possible impacts to the Pine Snake population dynamics.

Will the wider variations in temperatures impact the overall population dynamics of the Pine Snake?

How long will it take for impacts, if any, to become noticeable?

Reference

[1] Burger, Joanna and R. T. Zappalorti. 1988. Effects of Incubation Temperature on Sex Ratios in Pine Snakes: Differential Vulnerability of Males and Females. *The American Naturalist*. 132(4): 492-505.

SCUDEM III 2018 - Problem C - Snakes in the Long Run

With respect to the evolutionary pressures associated with greater climate variability, describe how your model would change and what changes might you expect to see in how the species will adapt.