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STUDENT VERSION
Solid Particle Erosion

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STATEMENT

The process of erosion is familiar to most of us from Earth science, where water or wind removes material from one location and transports it to another. However, it is also an important industrial process. We can think of erosion more generally as solid particles, entrained by a fluid, attacking and removing material from a surface. This process can adversely affect the performance and life of such equipment as pipes, turbines, and helicopter rotorblades. It is also the cause of the corona effect, sometimes called the Kopp-Etchells effect, experienced by helicopters where small pieces of eroded metal can oxidize and illuminate the rotorblades - a dramatic, but obviously undesirable condition for military operations. On the other hand, erosion can have useful applications such as sand blasting, abrasive deburring, and drilling hard materials. For a process of such significance, both good and bad, an understanding of the basic physical processes involved is required to take measures to either mitigate or take advantage of its effects.

In this project we will develop a mathematical model of the cutting process by which a relatively rigid particle removes material from a ductile surface. The canonical situation would be a piece of sand striking a metal surface. The mathematical model will be in the form of a system of differential equations resulting from application of Newton's Second Law of Motion. Our model is based on the first published attempt at an analytical description of solid particle erosion [1]. From the solution we will be able to determine the conditions for maximum material removal, and the role played by the different physical parameters of the model. In particular, we will be interested in the angle of attack at which maximum material removal occurs.

PHYSICS BACKGROUND

The physics knowledge required to formulate this problem is covered early in any calculus textbook aimed at science and engineering students. So the concepts should be review, but taking the time to go through the details will be time well spent.

We start with kinematics, the subfield of mechanics that describes motion without concern for the causes of that motion. For bodies that are rigid and large enough to ignore quantum effects, kinematics amounts to accounting for the position and orientation (and their rates of change) of the body in space. Position is typically given as a vector specifying the coordinates of the Center of Mass (CM) of the body with respect to some fixed coordinate system. Orientation, also a vector, is most often quantified by specifying the rotation of a body about coordinate axes whose origin moves with the CM.

We will be idealizing our physical world with two dimensions, so the position vector will be a 2-tuple. If the body is to remain in the plane, orientation will be reduced to a single number - the angle of rotation about an axis perpendicular to the plane of the body and passing through the CM. Therefore, the kinematics of our rigid erodent (the body causing the material removal) will be quantified with just three numbers, (x, y, ϕ), two representing translation in the plane, and the third describing its rotation about the CM, see Figure 1.

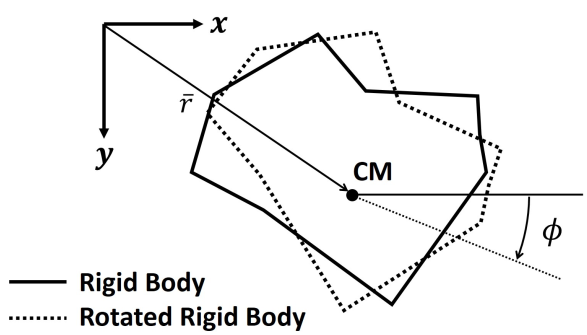

Figure 1. The position $\bar{r}$ of a rigid body in two dimensions and it's orientation ϕ with respect to an axis through the CM and perpendicular to the page

The second subfield of mechanics is kinetics. Kinetics relates applied forces to the motion they cause. Newton's famous three laws of motion are the foundation of Kinetics. Specifically, Newton's Second Law states that the vector sum of the applied forces acting on a particle is equal to the particle's rate of change of momentum. For many problems the mass of the particle under consideration is constant (or close enough), so that the rate of change of momentum is equal to mass times acceleration.

$$\sum \bar{F} = \frac{d}{dt}(m\bar{v}) = m\bar{a} \tag{1}$$

The direct application of Newton's Second Law will give us a vector differential equation relating the acceleration of the CM to the forces acting on the body. If we let F_x and F_y denote the sum of the force components in the x and y directions, respectively, then the component equations are:

$$F_x = m\ddot{x} \tag{2}$$

$$F_y = m\ddot{y} \tag{3}$$

where overdots are used as a short-hand for time derivatives, in this case the second derivative.

To complete the model we need to supplement these equations with a relationship that allows us to solve for the orientation of the body. Toward this end, consider a coordinate system anchored at the rigid body's CM, and partition the rigid body into a finite number of subsets. Consider the external force $d\bar{F}_i$ applied to the i^{th} subset of the partition. Let the mass of of the i^{th} subset be denoted dm_i (we will use dm_i to refer to the i^{th} subset itself, and its mass). First, consider dm_i as a particle and apply Newton's Second Law, $d\bar{F}_i = dm_i\bar{a}_i$. Now, take the cross product of each side of this equation with the position vector $\bar{r}_i$ that locates dm_i with respect to the coordinate system at the CM.

$$\bar{r}_i \times d\bar{F}_i = \bar{r}_i \times dm_i\bar{a}_i \tag{4}$$

Since the body is rigid, the subset dm_i will remain at a constant distance from the CM. As the rigid body rotates its orientation changes. From the point of view of the CM, this will look like the subset dm_i is moving along a circular path centered at the CM, see Figure 2. This suggests a convenient decomposition for the vectors $d\bar{F}_i$ and $\bar{a}_i$ with components normal, and tangent to the trajectory of dm_i. We denote the components with superscripts n and t, see Figure 3.

The computational convenience of this decomposition is that the cross product of parallel vectors is zero. Since the normal components are always parallel to $\bar{r}_i$, their cross products with $\bar{r}_i$ vanish.

$$\bar{r}_i \times \left(d\bar{F}_i^n + d\bar{F}_i^t\right) = \bar{r}_i \times dm_i\left(\bar{a}_i^n + \bar{a}_i^t\right) \tag{5}$$

$$\bar{r}_i \times d\bar{F}_i^t = \bar{r}_i \times dm_i\bar{a}_i^t \tag{6}$$

This is still a vector equation; however, the vectors that we are crossing lie the plane of the two dimensional rigid body. We know that the direction of their cross product will be perpendicular to the page. With the direction known (and unchanging throughout the motion) we need only find the magnitude of the vectors involved. We will use the notation that a vector's magnitude is denoted with the same symbol as the vector, but without the overbar. In general, the magnitude of a cross product is $\left|\bar{a} \times \bar{b}\right| = ab\sin\theta$, where θ denotes the angle between the vectors $\bar{a}$ and $\bar{b}$. Due to our choice of decomposition, the cross products only involve vectors at right angles, therefore since $\sin(90°) = 1$ their magnitudes satisfy

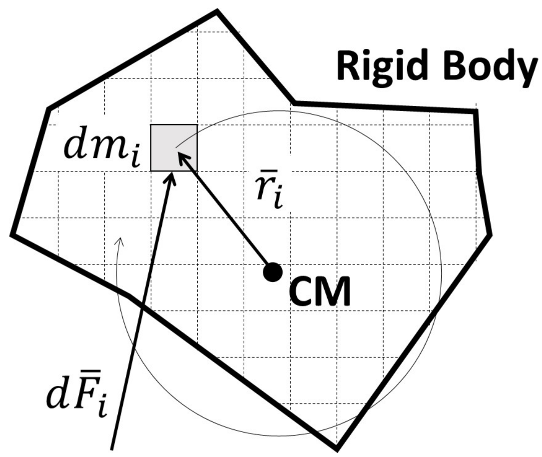

Figure 2. The force $d\bar{F}_i$ applied to the subset dm_i of the partitioned rigid body. As the body rotates, the subset dm_i will remain at a fixed distance from the CM; that is, viewed from the CM, it will travel in a circle.

$$r_i dF_i^t = r_i dm_i a_i^t \tag{7}$$

The product on the left hand side of this equation is commonly known as *torque*, and we will denote it by $d\tau_i$. On the right hand side we have the tangential acceleration. This is the rate of change of the magnitude of dm_i's velocity vector (the normal component of the acceleration, which does not play a role for our analysis, is due to the change in the *direction* of dm_i's velocity vector). With respect to the CM-based coordinate system, dm_i is traveling in a circle, denoting its angular displacement by ϕ_i the magnitude of its velocity is $r_i\dot{\phi}_i$. Furthermore, since r_i is constant

$$d\tau_i = r_i dm_i \frac{d}{dt}\left(r_i\dot{\phi}_i\right) = r_i^2 dm_i \ddot{\phi}_i \tag{8}$$

There is one more point to make about this equation for the subset dm_i. Since the body is rigid, each point will rotate with the same angular displacement, thus ϕ_i is the same for all i. So we can drop the subscript and refer to ϕ as the angular displacement, or rotation, of the entire body.

Now, sum over the partition:

$$\sum_i d\tau_i = \ddot{\phi}\sum_i r_i^2 dm_i \tag{9}$$

As we refine the partition $i \to \infty$ and $dm_i \to 0$, and the sums become integrals over the rigid body

$$\int\limits_{body} d\tau = \ddot{\phi}\int\limits_{body} r^2 dm \tag{10}$$

The integral on the right hand side of the equation has a special name, the *mass moment of inertia* of the body, typically denoted by I. It encodes not just the mass of the rigid body, but

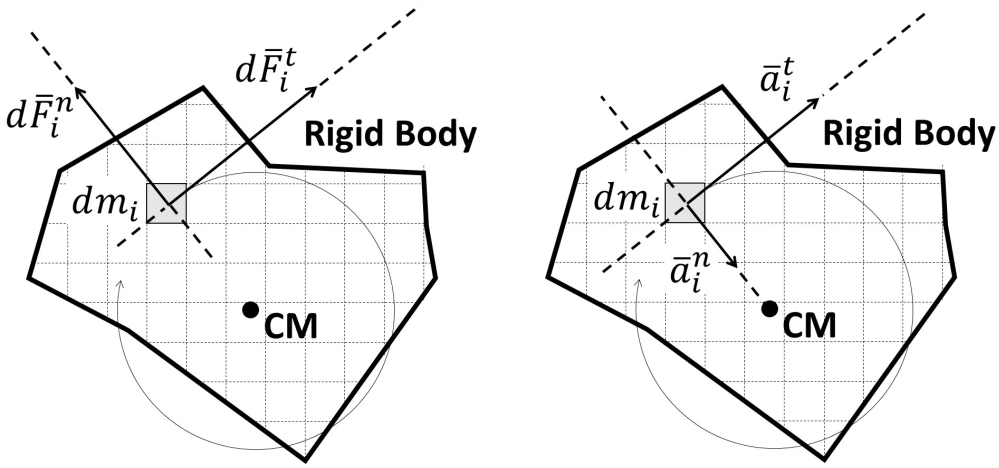

Figure 3. The components of force and acceleration that are normal, and parallel to the position vector from the CM to the subset dm_i.

how that mass is distributed with respect to the origin of the coordinate system. We can simplify the last expression by substituting τ for the total torque applied to the body, and I for the mass moment of inertia:

$$\tau = I\ddot{\phi} \tag{11}$$

The three equations are now established. The first two are the component equations of Newton's Second Law, relating the x and y acceleration of the CM to the forces applied to the rigid body. The third equation relates the rotation to the applied torque. Now we are prepared to model the trajectory of a rigid erodent attacking a ductile surface.

EQUATIONS OF MOTION

To estimate the amount of material removed from the surface, we will use the trajectory of the cutting tip of the erodent. Since the erodent is assumed rigid, the position of the cutting tip is calculable from the knowledge of the x and y coordinates of the CM, the tip's distance r from the CM, and the body's rotation ϕ. Therefore, we can determine the trajectory for the tip by solving the differential equations (2, 3, 11) for x, y, and ϕ. We only need to account for all of the forces (and torques) applied to the erodent during its motion.

Figure 4 shows an arbitrary erodent at the moment of contact with the ductile surface. The ductile surface is assumed stationary - even though in practice it may be in motion, e.g. turbine blades. By assuming the ductile surface is stationary we are observing the impact as if we were located on the ductile surface. The angle of the impact α (the angle of attack) is measured from the horizontal; a grazing impact would have $\alpha = 0$, and $\alpha = \pi/2$ for a normal impact. The speed of the impact is v_0.

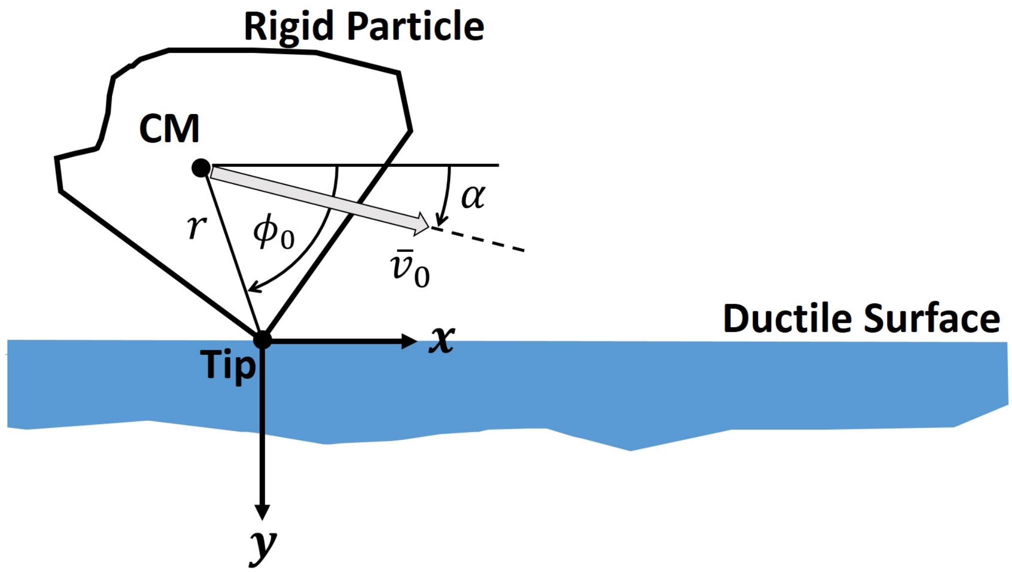

Figure 4. The initial conditions of the impact of the erodent upon the ductile surface.

The only force acting on the erodent is contact with the surface material, see Figure 5. Forces due to electromagnetic interactions, air resistance, gravity, etc. are all negligible compared to the contact force. To simplify the analysis we assume that the surface material is perfectly plastic, so that when the surface material is disturbed it is in a state of hydrostatic stress. That is, when the erodent is cutting through the perfectly plastic surface material, the surface material will exert a constant pressure on the erodent that will act normal to the contact area between the two materials. This pressure is called the *flow stress*. It is a material property of the surface material and we will denote it by σ. Therefore, the force acting on the erodent has magnitude σA, where A is the area of contact between the materials; the direction of the force is normal to the contact surface and opposes the motion. To simplify our derivation we will assume that the erodent has a uniform thickness b out of the plane of motion. Therefore, the contact area is $A = bh$, see Figure 6. The magnitude of the total force on the erodent is $F = \sigma bh$.

We first derive an expression for the x component of the force. Consulting with Figure 6, $F_x = -\sigma bl$; the minus sign on the right hand side is consistent with our coordinate system.

Ahead of the erodent the surface material builds up into a lip so that the contact area between the erodent and surface extends above the undisturbed height of the surface, Figure 6. We will assume that the height l, which takes into account this lip, is proportional to the depth of the tip y_{tip} with constant of proportionality ψ. Thus, $F_x = -\sigma b\psi y_{tip}$, and so

$$m\ddot{x} = -\sigma b\psi y_{tip} \tag{12}$$

This is unsatisfactory because it introduces a new variable, y_{tip}, but provides no additional

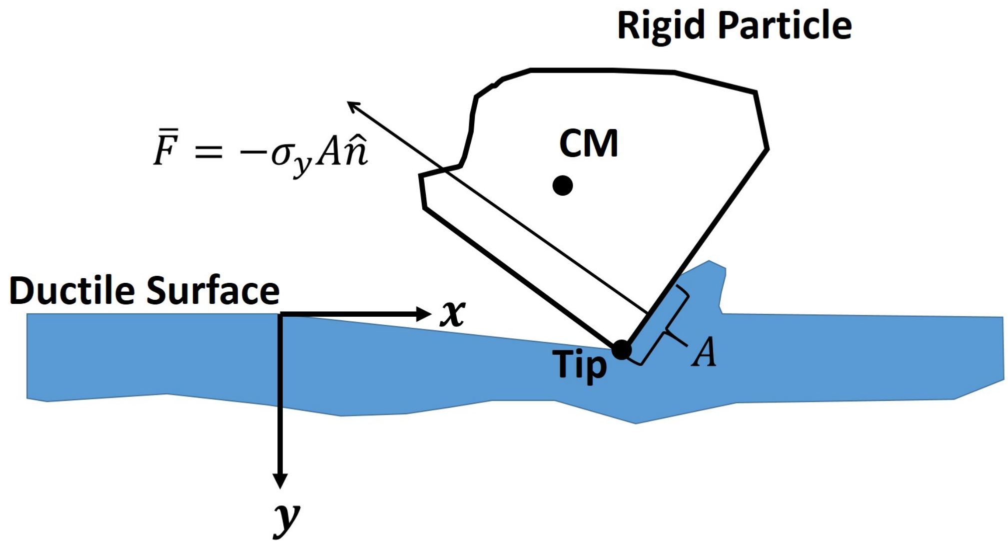

Figure 5. The contact force acting on the erodent opposes the motion and is normal to the contact surface.

equations. Therefore, to solve our system we will need to find an additional equation, or relate y_{tip} to the preferred kinematic variables x, y, and ϕ. The most convenient approach is to replace y_{tip} with the other kinematic variables through a careful examination of their meaning. First, we need to distinguish between *translation* and *position*. The position of any point of the body is its location in the plane, it is the composition of a translation and rotation. Second, in a rigid body all points experience the same translation (each point just has a unique starting point), and the common translation is x and y of the CM. So to find the position y_{tip} we find y starting from the tip's initial position (the origin), and then apply the rotation about the CM. Note that we use the initial condition $\phi(0) = \pi/2$ to evaluate the contribution from rotation; this will be explained when we consider the equation for ϕ. Thus, we now have,

$$m\ddot{x} = -\sigma b\psi\left(y - r\sin\phi\right) \tag{13}$$

For the y equation we assume that the y component of force is proportional to the x component, $F_y = kF_x$. This is a bold assumption that is justified by the resulting model's ability to predict observed patterns of erosion. It is certainly something that needs to be examined further, but not here. Thus,

$$m\ddot{y} = -k\sigma b\psi\left(y - r\sin\phi\right) \tag{14}$$

Finally, we tackle the ϕ equation (11). If r is the distance from the CM to the cutting tip of the erodent, then the torque applied to the erodent by the surface material is approximately $\tau \approx -rF_x$ (negative because a positive F_x would induce a negative rotation in our system). Implicit in this expression for the torque are some assumptions and simplifications that we will explain below.

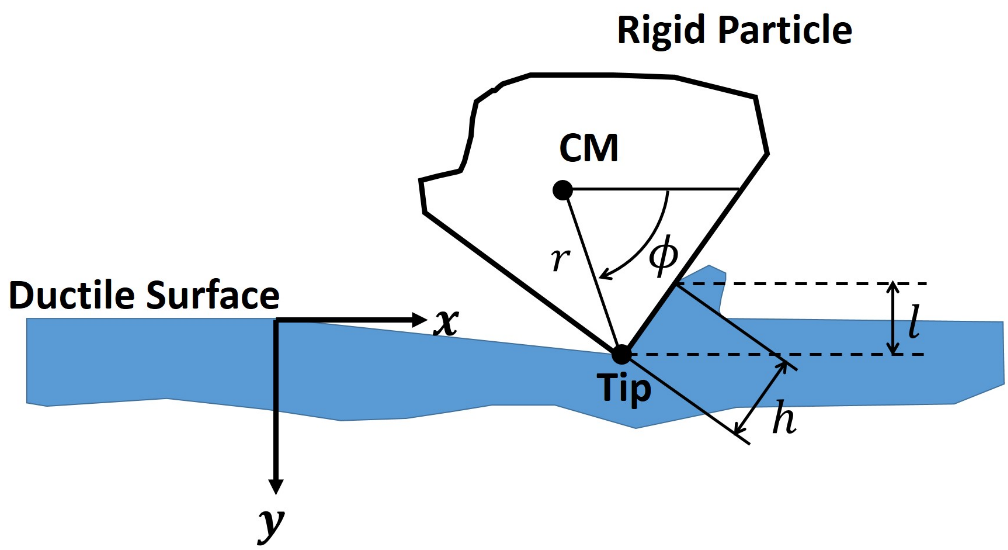

Figure 6. The contact area between the erodent and surface is $A = bh$, therefore the magnitude of the contact force is $F = \sigma bh$. When considering the x-component only $F_x = -\sigma bl$.

- Even though the torque is the result of a contact force distributed over the entire contact surface, we will approximate the area of contact with a single point.
- The geometric center of the contact area would be the ideal point of application (since the contact force is uniform over the contact area), but for convenience we choose r - the distant edge of the contact region (see Figure 4). Assuming that the contact area is small compared to the size of the erodent then this should not introduce significant error into our model.
- The y component of the force does not contribute to the torque. This is equivalent to assuming that the cutting tip starts directly below the CM, $\phi(0) = \pi/2$, and remains there. Thus, the y component of the force passes through the CM and contributes no torque. This assumption is based on the reasonable expectation that the cutting tip will be ahead of the CM in some impacts and behind it in others, but *on average* the cutting tip will be directly below the CM.

We also need the mass moment of inertia I of the erodent. As a representative of a typical mass moment of inertial we choose that of a flat cylinder of radius r and height b. Note that we are not assuming that this is a representative shape (no tip!), but that this mass moment of inertia is representative. For such a shape the mass moment of inertia is $I = mr^2/2$.

All of these assumptions and approximations result in the ϕ equation:

$$\frac{mr^2}{2}\ddot{\phi} = r\sigma b\psi y \tag{15}$$

To complete the model we need initial conditions for each equation. The initial conditions for the translation (x and y) are the initial position of the cutting tip (the origin), and the initial translational velocity components (the components of $\bar{v}_0$). The intial rotation is the average value $\phi(0) = \pi/2$, and we will leave the initial rotational velocity as an undetermined constant $\dot{\phi}_0$ for now. In summary:

$$m\ddot{x} = -\sigma b\psi\left(y - r\sin\phi\right) \tag{16}$$

$$x(0) = 0 \quad , \quad \dot{x}(0) = v_0\cos\alpha \tag{17}$$

$$m\ddot{y} = -k\sigma b\psi\left(y - r\sin\phi\right) \tag{18}$$

$$y(0) = 0 \quad , \quad \dot{y}(0) = v_0\sin\alpha \tag{19}$$

$$\frac{mr^2}{2}\ddot{\phi} = -r\sigma b\psi y \tag{20}$$

$$\phi(0) = \pi/2 \quad , \quad \dot{\phi}(0) = \dot{\phi}_0 \tag{21}$$

Linearized Equations

It is not far-fetched to fisrt examine what happens if the erodent rotates very little during the cutting process. Since $\phi(0) = \pi/2$ this would amount to considering the model when ϕ remains small throughout, that is $\sin\phi \approx 0$. The relevant system of equations becomes:

$$m\ddot{x} = -\sigma b\psi y \tag{22}$$

$$x(0) = 0 \quad , \quad \dot{x}(0) = v_0\cos\alpha \tag{23}$$

$$m\ddot{y} = -k\sigma b\psi y \tag{24}$$

$$y(0) = 0 \quad , \quad \dot{y}(0) = v_0\sin\alpha \tag{25}$$

Note that this assumption had two consequences; first, it uncoupled ϕ from the x and y equations as well as made the ϕ equation irrelevant (why do we need a differential equation to model what is essentially a constant?). Second, it renders the equations for x and y linear, making their solution considerably simpler. It is this linearized system that will be used to answer the questions in this project.

PROJECT

Our ultimate goal is to determine the relationship between the angle of attack α and the volume of surface material removed and how this relationship is affected by the paramters in our model. Therefore, we will have quantitative information that we can use to deploy mitigating material if our goal is to minimize the effects of erosion. Or, in the case that we wish to use erosion to accomplish some desired result, such as removing rust from a ferrous surface material, then we can know how best to direct a stream of erodents to maximize material removal.

To begin, let's establish some values for the constants characterizing the erodent and surface material. Table 1 contains numbers that are typical for a quartz sand erodent and an aluminum surface material - with the exception that σ is almost four times larger than a typical flow stress for aluminum. Finnie [1] attempts an explanation of this, proclaiming that strain rate effects (which raise σ for high speed events), and other factors not considered in this simplified model account for the enlarged flow stress. Note that the units of mass are quite unusual. *Dozens of Slugs* is the natural unit of mass in the English system with inches as the length unit and seconds as the time unit.

Parameter	Value	Units
k	2	dimensionless
ψ	0.04	dimensionless
r	0.04	in
b	0.04	in
m	2.855×10^{-7}	dozens of slugs
v_0	4400	in/s
σ	200×10^3	lb/in^2
α	$20 \times \frac{\pi}{180}$	radians

Table 1. Model Parameters

Question 1: For the parameters provided, what are the solutions $x(t)$ and $y(t)$?

Notice that even though the system is coupled, the y equation can be solved independently by traditional methods, and its solution substituted into the x equation for its subsequent solution.

Hint: Equation (24) is uncoupled from equation (22), so it can be solved by itself. Once the solution $y(t)$ is known equation (22) can be solved by integrating twice and using the intial conditions to determine the arbitrary constants.

As a sanity check, Figure 7 shows the time histories of x, y, $\dot{x}$, and $\dot{y}$; Figure 8 shows the trajectory of the particle tip parameterized by $x(t)$ and $y(t)$.

Question 2: For the parameters provided, when does the cutting stop?

When we make our calculation of the volume of material removed we will need to calculate the area between the tip's position and the positive x-axis. The solution to the differential equations (22, 24) provide a parametric representation of the tip's trajectory. So the missing information needed to calculate the volume is when to stop our simulation. That is, what is the time t_c when the tip stops cutting? There are two conditions that will end the cutting.

1. The erodent rebounds off of the surface, that is $y(t_c) = 0$ for $t_c > 0$.

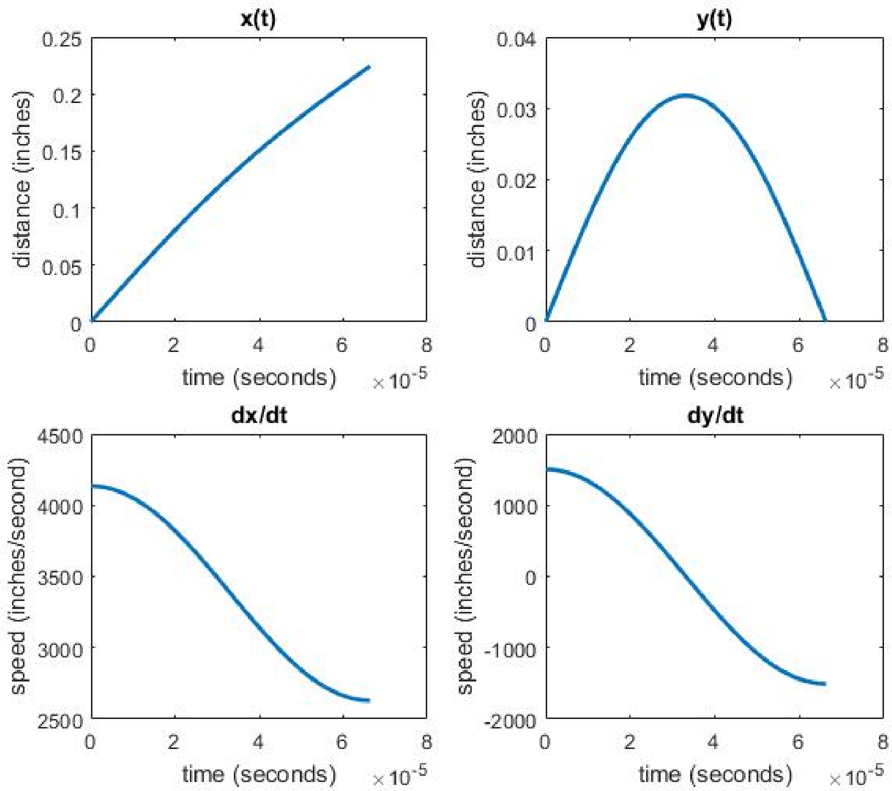

Figure 7. The x and y components of position and velocity for the particle tip.

2. The erodent loses all of its forward momentum before it can rebound, that is $y < 0$, but $\dot{x}(t_c) = 0$ for $t_c > 0$.

Whichever condition occurs first will determine t_c and end material removal.

Hint: Figure 9 shows the candidate t_c values as the graphs of $0 = y(t_c)$ and $0 = \dot{x}(t_c)$. The actual value of t_c will be the lesser of the two candidates, indicated by a bold curve in the figure. Even though these are smooth functions there is a cusp in the graph of equation $0 = \dot{x}(t_c)$. This is because the figure plots only the real part of the complex solution to the equation - only when the solution is purely real (no imaginary part) is it of physical significance. It transitions from a complex to a real solution precisely where the two curves intersect.

Question 3: For the parameters provided, what is the volume removed?

If we know the trajectory of the tip $(x_{tip}(t), y_{tip}(t))$ then we could integrate to find the area between the trajectory of the tip and the x-axis. This area multiplied by the width of the erodent, b, is the

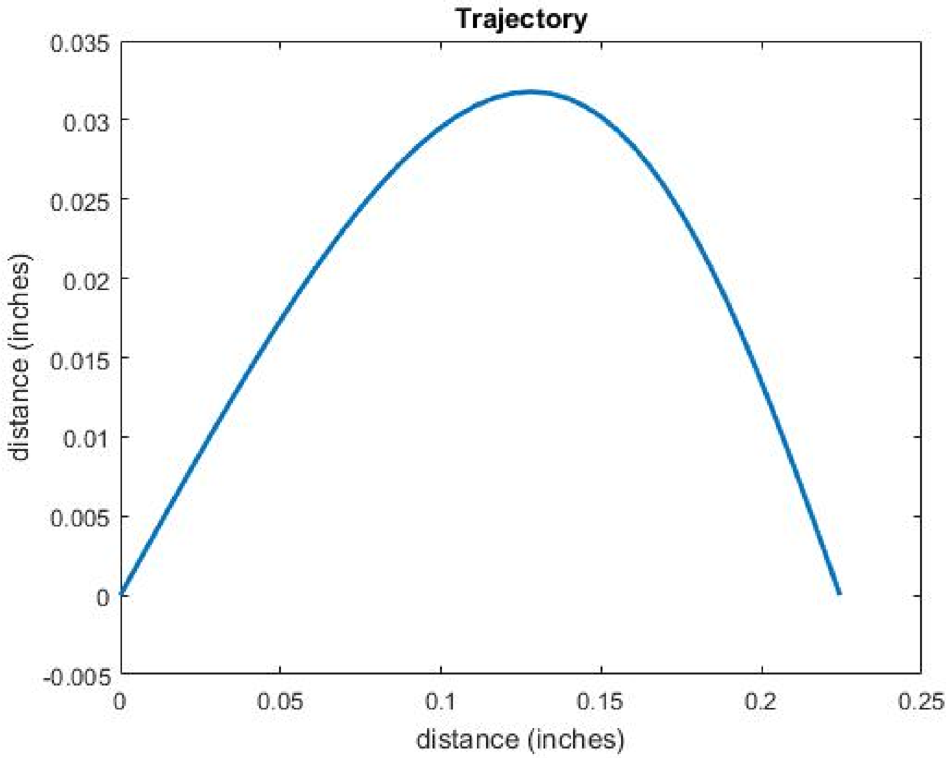

Figure 8. Trajectory of the particle tip.

volume of removed material. Since the linearized model ignores rotation of the erodent, we can use $x(t)$ and $y(t)$, starting at the origin, as surrogates for $x_{tip}(t)$ and $y_{tip}(t)$

$$vol = b\int_{t=0}^{t_c} y\,dx \tag{26}$$

$$= b\int_{0}^{t_c} y\dot{x}\,dt \tag{27}$$

Hint: I recommend using the expressions for y and $\dot{x}$ found in Question 1, and the value of t_c found in Question 2 to perform a numerical integration using something like midpoint rule.

Question 4: For the parameters provided (except α), at what angle is the volume removed maximized?

Hint: The angle at which maximum material removal occurs, α_{max}, will happen when

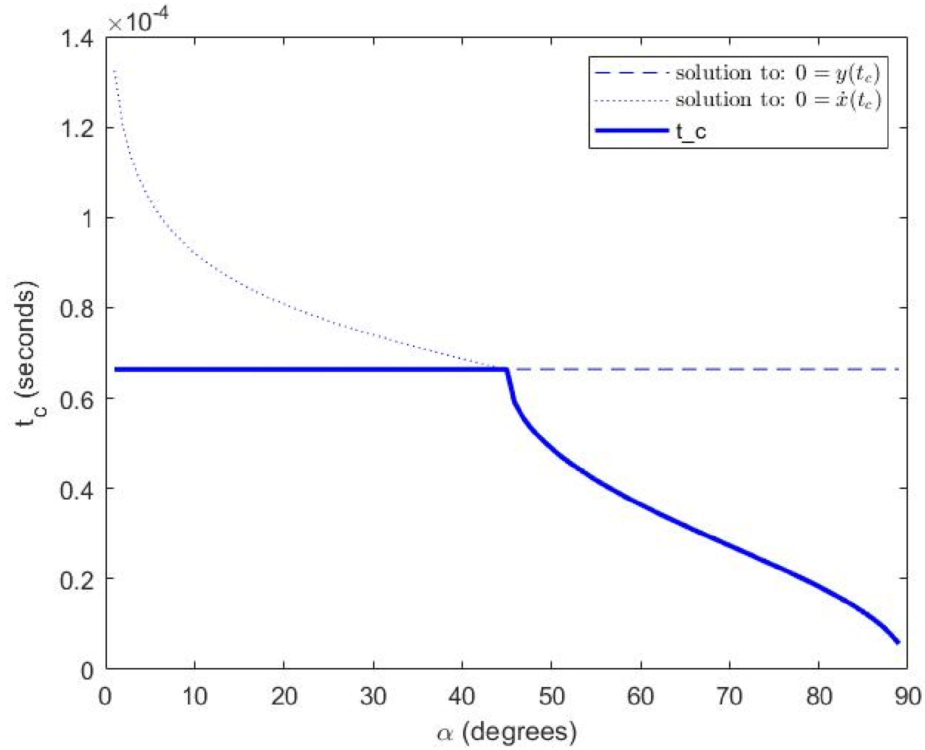

Figure 9. The value of t_c (bold): Where t_c is constant cutting ends because the erodent has rebounded off of the surface, the other values of t_c occur when the erodent has lost its forward momentum.

$$0 = \frac{d}{d\alpha}\left(vol\right)\bigg|_{\alpha=\alpha_{max}} \tag{28}$$

I highly recommend finding α_{max} numerically. That is, repeat the calculation for the volume removed at *many* values of α between 0 and $\pi/2$ (note that $\alpha > 0$ is required).

Question 5: Does the erodent's speed v_0 affect the angle of maximum material removal?

We implicitly assume that v_0 is large enough to remove material - that is, the stress produced from the impact is large enough that the surface material will *flow*. If we maintain that this assumption is true for any v_0, will different values of v_0 change the angle of maximum material removal? Put differently, does the initial velocity v_0 have an influence on the volume of material removed during an impact?

Hint: Select (at least) 3 different initial velocities that cover a relatively large, but realistic range, say 2000 to 8000 in/s. On the same graph plot the volume removed vs. angle of attack for each initial velocity. From these graphs is should be clear if the speed is effecting angle of maximum material removal.

Question 6: Does the choice of erodent affect the angle of maximum material removal?

The erodent is characterized by two parameters, its size r and its mass m. Within any real stream of particles we can expect a considerable amount of variability in these two numbers, they could even change mid-stream if particles collide and break-up into smaller particles. So measuring the angle of maximum removal from a stream of particles instead of a single particle impact could potentially mislead our conclusions if r and m have an effect of the angle of maximum material removal. Will either of them have a major impact on our results?

Hint: First, consider the impact of r. The equations that govern the material removal are equations (22, 24). Is r even present in these equations? If not, then it cannot affect the results. Second, consider the mass m. As in Question 5, pick (at least) 3 masses that would be typical for small particles such as sand (use the mass value in Table 1 as a guide for a typical mass). On a single graph plot the volume of material removed vs. the angle of attack and compare the angle at which maximum material removal occurs for each mass.

Question 7: Does the surface material affect the angle of maximum material removal?

The surface material is characterized only by its flow stress σ. The flow stress for other ductile materials that fit our canonical situation are shown in Table 2.

Metal	Flow Stress	Units
Mild Steel	37×10^3	lb/in^2
Brass	27×10^3	lb/in^2
Copper	10×10^3	lb/in^2
Stainless Steel	75×10^3	lb/in^2
Titanium	136×10^3	lb/in^2

Table 2. Flow Stress

Hint: On a single graph plot the volume of material removed vs. the angle of attack for 3 (or more) of the materials in Table 2 to determine if the surface material has an effect.

Question 8: How do the modeling parameters ψ and k affect the angle of maximum material removal?

Recall that ψ is the constant of proportinality between the depth of the eroding tip, and the height of the material building up in front of the erodent. It's value is required to calculate the area of contact between the erodent and the surface material, it's value was *assigned* to be 0.04. There was no physical justification given for this value - it was more wild guess than educated guess.

Similar to ψ, k is a constant of proportionality. This time, between the x and y components of force acting on the erodent. That this ratio between the force compenents remains constant is a bold assumption; that the force acting on the erodent does not change direction during the cutting process. And like ψ, it's value was guessed ($k = 2$).

So, were the guesses made for the values of ψ and k consequential in determining α_{max}?

Hint: As in Questions 5, 6, and 7, we need to select some representative values for ψ and k then plot the volume of material removed vs. angle of attack. Selecting typical values of these parameters, however, is not as straightforward as with the physical quantities of velocity, mass, radius, and flow stress. Therefore, I recommend the following: For ψ use positive values and stay less than 5.0 (this is more than 100 times our initial choice for ψ). For k, remember it is the ratio of the y-component of force to the x-component. So k should also be positive, and somewhere "close" to unity, $0.1 < k < 10$.

Additional questions That are considerably more difficult:

There are three ways that this investigation could go further, each addresses our biggest assumptions in deriving the linearized model.

1. What if we do not make the assumption that ϕ remains small throughout the cutting process? Will this change our answer for the angle of maximum material removal? Note that this requires making an additional assumption regarding the initial angular velocity of the erodent $\dot{\phi}(0)$.
2. We assumed that that $F_y = kF_x$ with k constant, independent of α. A first step towards a more more realistic model would be that the force acts in the direction opposite of the erodent's intial velocity, $k = \tan\alpha$. This will allow us to keep the linearized system of equations while reducing the severity of one of our assumptions. Will this change our answer of the angle of maximum material removal?
3. Addressing the k assumption again, what if k changes with the trajectory of the erodent? A better expression for k would produce a contact force opposing the velocity of the projectile throughout the motion, not just its initial direction. So, what if

$$\frac{F_y}{F_x} = \frac{\dot{y}}{\dot{x}} \text{ ?} \tag{29}$$

Under this new expression for k even the two equation system neglecting ϕ is nonlinear. Will this change our answer for the angle of maximum material removal?

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