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A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
Employee Attrition

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STATEMENT

A large online retailer has a problem. It hires 100,000 "last mile" delivery workers to deliver its packages. However, these part-time employees leave the company at a rate of 9% per month. We will construct a model that predicts the number of employees in this job during a given month.

Part 1: Losing Employees

A. Make a list of every factor you can think of that will affect the employee population change. We will assume no new hires for now.

B. Using the 5% rule, determine which factors should be included in the model and which can be assumed to be insignificant. Label each factor from part A. as either A for assumed to be small or M for included in the model.

> **The 5% Rule**: If you believe the factor will effect the model by 5% or more, it must be included in the mathematical formulation of the model. If you believe the effect will be less than 5%, list the factor as an assumption, i.e. assume the effect is minimal.

C. Formulate a mathematical model (an equation) for the population of employees. Use $A(t)$ to represent the population. Be sure to include all of the model factors from part B.

D. What initial condition will you use?

E. Predict the behavior of your model. What will happen in the second month? What will the behavior be for large times?

F. Use the `DSolve` command in Mathematica to find the solution function for your model's Initial Value Problem (IVP). We would like to store the resulting function to use later. The syntax for this is

```
A1[t_] = A[t] /.  First@DSolve[{A'[t]==????,A[0]==????},A[t],t]
```

G. Plot the resultant function to check your answer.

H. Does your result match your prediction from part E?

I. Find the population at time $t = 120$ months. What is wrong with this prediction? What assumption did we violate?

Part 2: Stock Options

The workforce will only support the hiring of 5,000 employees per month, so a new strategy is employed to retain workers. In addition to hiring 5,000 per month, 10% of the employees are offered stock options because it is believed that this will reduce the attrition rate to 3% (for these employees only).

A. What new factors might now affect the total population?

B. Again, divide these new factors into assumptions and model components.

C. Formulate a mathematical model (an equation) for this second population of employees, call it $B(t)$. A new model for $A(t)$ may also be necessary.

D. What initial condition will you use, $B(0)$?

E. Predict the behavior of your model.

F. Find the solution functions for your model for $A(t)$ and $B(t)$.

G. Plot the resultant functions to check your answer.

H. Does your result match your prediction?

I. What is the end behavior for the total employee population, i.e. $A(t) + B(t)$ for $t \to \infty$?

Part 3: Achieving Balance

The plan to offer stock options to 10% of the employees has succeeded, but too well. We would like to reach exactly 100,000 employees in the steady state.

A. What percentage of the employee population should be offered stock options in order to maintain this level?

B. What are the resultant functions at this stock option level?

C. Plot both population levels and the total population on the same graph.

D. Offer a summary recommendation to the company based on your work.