Engaging and supporting students as they explore mathematical modeling in differential equations

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Background

- Emmanuel College (Boston, MA) and University of Redlands (CA) are liberal arts institutions
- 2000-level courses for math majors and minors
    - Differential Equations
    - Mathematical Modeling
- 10 - 25 students per class

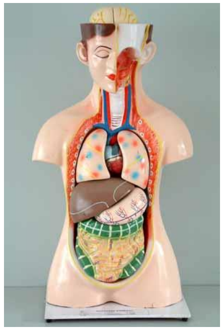
Modeling projects

Goal: apply techniques learned in class to construct and analyze mathematical models involving differential equations.

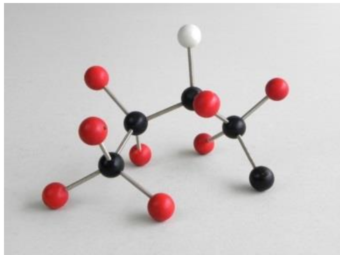
- Several group projects are given in a semester.
- Students choose a topic for the final project.
- Submit written report.
- For the final project, students also present at a poster session.
- Weekly workshop/recitation time focused on group work.

SUPPORTING STUDENTS

What is a model?



"A model is a purposeful representation"



– Tony Starfield

***Math* modeling** is the art of using mathematics (equations, graphs, etc) to represent and explain the behavior of the world around us.

Guidelines for project report

- Provide example reports from previous semesters
- Provide a peer-reviewed research paper
- Provide templates

Project Checklist

Projects are graded out of 100 points. Please read the following information carefully and only check off a box when you are sure the project has achieved those goals.

- ☐ The solution to the project is a typed report, single spaced, 12pt font. The project is not more than 6 pages and you have condensed into it only the most important information. (5 points)
- ☐ The final report has been proof read by every member of the group. You are handing in work that you are proud of and is of professional quality! There are no grammar or spelling errors in the paper. (10 points)

  - **Discussion** – This is a discussion about what you found. What do the results as a whole tell you? What do they mean or how do they answer your original question? Are there other possible models that might work better or worse than yours? Were there any BIG assumptions made that should maybe be reassessed?
  - **Conclusion** – This is a very short review of what you did in the paper. A reader should be able to read just your introduction and your conclusion and get the basic idea of what your model was, what results were, and why they matter.

- ☐ The **mathematics is clearly stated in words** along with equations so that another math student can understand it. Please EXPLAIN the equations. What do they mean in terms of the system being modeled. (15 points)

Rubric (50 pts)

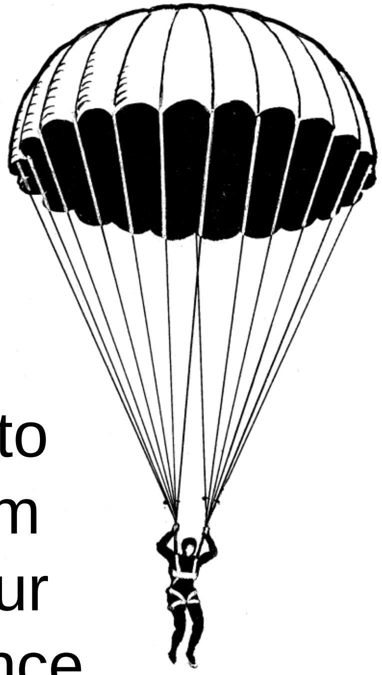
- Presentation (5 pts)
  - Neatness counts; proofread work
- Clarity (10 pts)
  - Is the problem, model, solution details, and interpretation clear to the reader and easy to follow?
  - Mathematics is clearly stated in words along with equations
  - Graphs are labeled and referenced
- Mathematical content (35 pts)
  - Solution and analysis are correct (check units)
- The “it” factor
- Extra point awarded if typed using LaTeX

DEVELOPING PROJECTS

How to come up with new ideas

1. Look at an example online or in an old textbook.
2. Think about another system that behaves that way (when in doubt add zombies or something relevant)
3. Rewrite the problem with a modern storytelling approach to peak student interest.
4. Be prepared for a bit of chaos the first time you try a new problem...the students are bound to find weird solutions.
5. Scaffolding of projects is helpful – first project gives lots of information & support

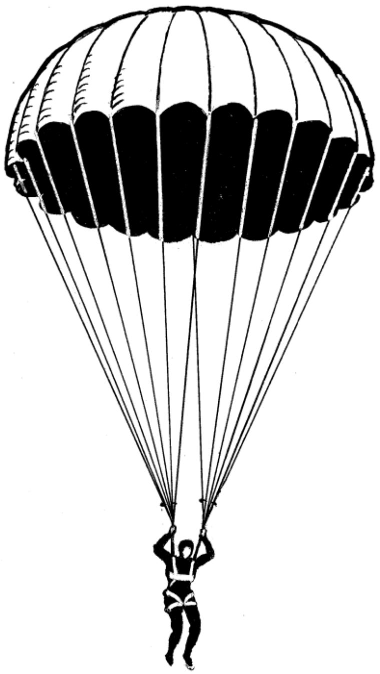
Project 1: Parachuting



Problem: You are part of the elite zombie containment team and are training to parachute into the disaster zone. On your first jump, you drop from a helicopter and immediately pull the ripcord of your parachute. You fall towards Earth under the influence of gravity.

- Classic Newtonian physics problem.

Project 1: Parachuting



Given assumptions:

- Forces – gravity and air resistance
- Constants (mass, v_0)

Prompts

- Construct and solve IVPs for velocity and position.
- Construct a slope field.
- What is the terminal velocity?
- What if $v(0) = -1$? What if $v(0) = 3$? Discuss what these conditions mean physically. Determine what effect, if any, they have on your long-term solution by looking at your slope field.
- How long does it take him to reach the ground?
- What is your velocity when you hit the ground? Will you survive the landing?
- What happens if the chute does not open?

Project 2: Zombie Apocalypse

Problem: Ten zombies rose from the grave today. How long before every Bostonian is a zombie?

- Analogous to modeling the spread of disease (SI model).
- Adapted from Mathematical Modeling of Zombies by Robert J. Smith?

Project 2: Zombie Apocalypse

- **Basic Model**: Consider constant zombie growth rate (# individuals per zombie per day)
- **Improved Model**: growth rate declines linearly with # of zombies
- **Attack Model**: How many zombies per day do we need to destroy to save Boston?
- **Analysis**: What if zombies are more/less aggressive? What if the initial number is larger/smaller? What are things can you explore?

Project 3: Medication

Problem: Return of the Zombies! Merck, the nearby pharmaceutical company, developed a pill to protect us from zombies. How long does it take the pill to take effect to grant us immunity?

- Model flow of medication through the body
- compartmental modeling / System of equations
- Adapted from Borelli and Coleman (2004)

Project 3: Medication

- Consider pill’s effect on healthy vs sick individuals (different rate constants)
- Determine the time at which maximum medication level occurs.
- Consider instantaneous dose vs dissolve continuously (different initial conditions)

Other Project examples

- Romeo and Juliet - (S. H. Strogatz, Love Affairs and Differential Equations, Mathematics Magazine, 61, 1988)
- Circuit with on/off switch (Laplace Transforms)
- Richardson’s Arm Race Model
- Lead Poisoning in Flint (linear system)
- Difference equations:
  - Population dynamics of endangered species Jaguar
  - Testing possible theories for the extinction of the Neanderthals.

FINAL PROJECTS

How students find final projects

- Textbooks
- Websites
- Research projects
- Personal interests / major
- Pure imagination
- This can be a huge and uncomfortable challenge for some students

SCUDEM - SIMIODE Challenge Using Differential Equations Modeling

- Four students in spring 2018 course participated
- Chosen problem: Sorting Recyclables – is a method to separate paper using a fan feasible?
- Given option to use their SCUDEM model for final project
- For the final project, they were asked to improve and add to their model after feedback from SCUDEM Competition Day.

One student's progress

Project 2: Zombies

Z = zombies → $Z(t)$ = zombies after t days $Z(0) = 10$

α = zombie growth rate (# of individuals per zombie per day.

H = Total population of Humans in Boston $H(0) = 600{,}000$

1. • The initial value problem for # of zombies after t days is

$$Z' = \ln(\alpha) Z, \quad Z(0) = 10, \quad \text{1st order linear ODE}$$

$$\frac{dz}{dt} = \ln(\alpha) z$$

$$\int \frac{1}{z}\, dz = \int \ln(\alpha)\, dt$$

One student's progress

Part 2: Linearizing the System

Now that we have our equilibrium points, we can analyze them by linearizing the system. To do so, we find the Jacobian matrix of the nonlinear system of differential equations

$$J(H_E, L_E) = \begin{bmatrix} \alpha - \beta L_E & \beta H_E \\ \delta L_E & -\gamma + \delta H_E \end{bmatrix}$$

Evaluating the Jacobian matrix about the equilibrium point $(H_E, L_E) = (0,0)$ gives us

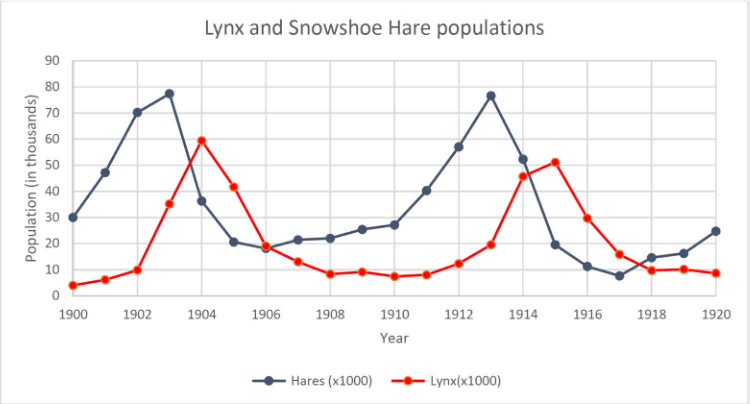
$$J(0,0) = \begin{bmatrix} \alpha & 0 \\ 0 & -\gamma \end{bmatrix}$$

This matrix as characteristic equation

$$(\alpha - \lambda)(-\gamma - \lambda) = 0$$

The eigenvalues and associated eigenvectors

$$\lambda_1 = \alpha, \qquad v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$



Student feedback from course evaluations

- *“I enjoyed the projects because they helped reinforce the subject material and see where this class is useful in the real world.”*
- *“I enjoyed how the course was not simply another math course, but several examples that we worked through made the course more practical. Moreover, these examples demonstrated how differential equations are applicable to engineering and medicine and other applications.”*
- *“Projects were a good way to connect applications of ODE to the actual mathematical formulation and solving”*
- *"I think about the world differently now - and feel somewhat capable to create models of the world around me."*
- *"Project based = actually understanding and using what we learn"*
- *"I love how this class is so applicable to the real world and all fields. It's relevance inspired me to work hard".*
- *“**Add more zombies**”*

RESOURCES

Resources: Textbooks

- Differential Equations and Boundary Value Problems by Edwards & Penney
- Elementary Differential Equations and Boundary Value Problems by Boyce & DiPrima
- Differential Equations by Borrelli & Coleman
- Introduction to Differential Equations with Dynamical Systems by Campbell & Haberman
- Topics in Mathematical Modeling by Tung
- A Course in Mathematical Modeling by Mooney & Swift
- MAA Writing Projects for Mathematics Courses: Crushed Clowns, Cars and Coffee to Go by Crannell, LaRose, Ratliff, and Rykken

Resources: Websites

https://simiode.org/

SIMIODE RESOURCES ABOUT SUPPORT COMMUNITY BLOG NEWSLETTER SCUDEM DONATE

Please use the Comments tab at each resource to make suggestions, point to corrections needed, relate experiences in your use of the resource, upload further resources we will add to the resource, discuss technical materials, and share your thoughts on the material. These comments will go to the author and the SIMIODE editorial leadership.

Modeling One — First Order Differential Equations +

Modeling Two — Numerically Solving Differential Equations +

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Modeling Three — Second Order Homogeneous Differential Equation Models +

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Modeling Four — Second Order NonHomogeneous Differential Equation Models +

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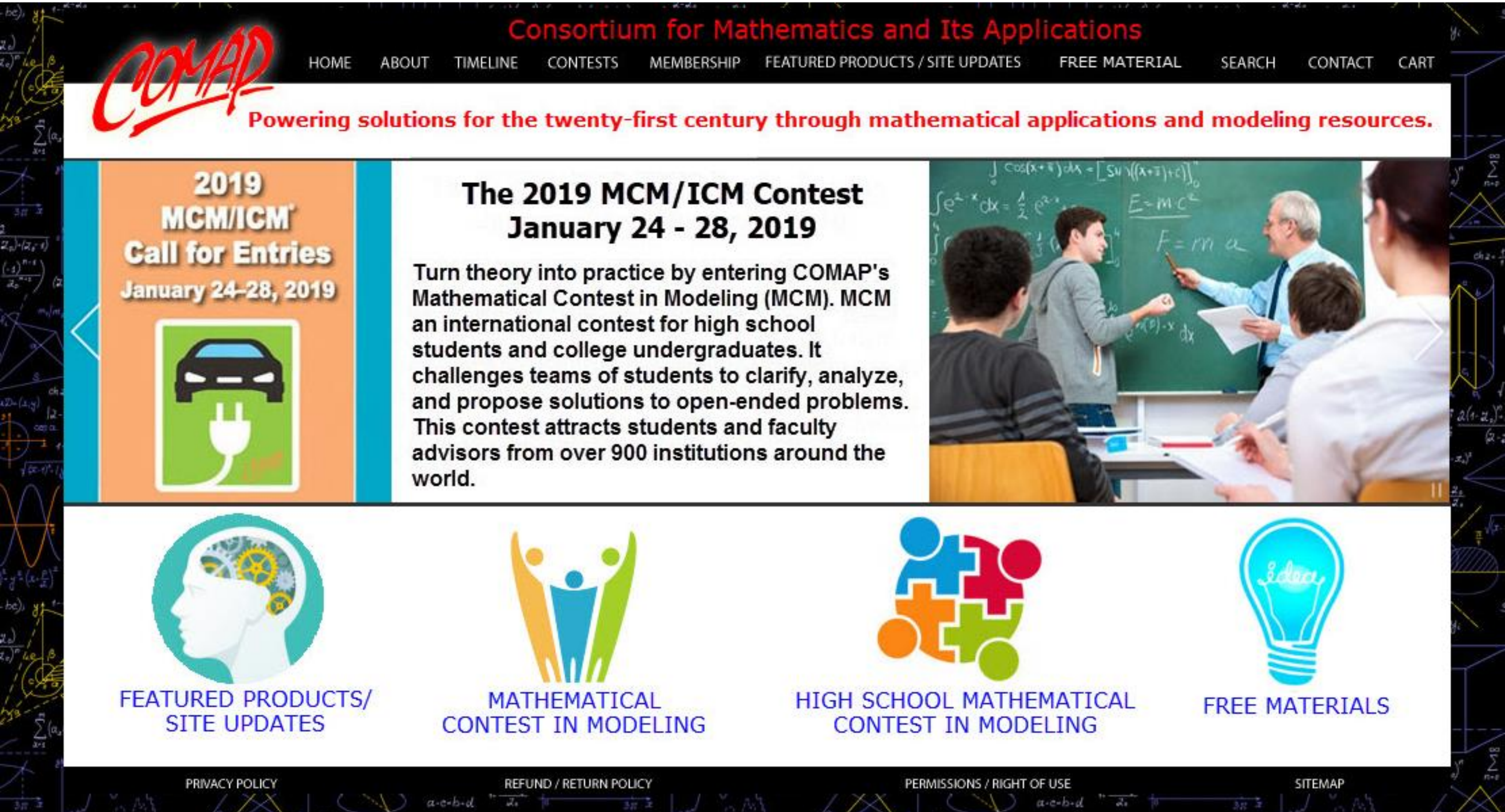
Modeling Five — Linear Systems of Differential Equation Models +

Resources: Websites

https://www.comap.com/



Resources: Websites

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Materials to Support the Teaching and Learning of Ordinary Differential Equations

- Printed and Print-Related Materials
- Software

Printed and Print-Related Materials

Inquiry-Oriented Differential Equations (IODE)

IODE is a first course in differential equations focused on understanding of the big ideas in first order, second order, nonlinear, and systems of differential equations, taught using an inquiry-oriented approach. The course is designed as a full semester course and topics covered include solving ODEs; numerical, analytic and graphical solution methods; solutions and spaces of solutions; linear systems; linearization; qualitative analysis of both ODEs and linear systems of ODEs; structures of solution spaces.

UMAP Journal

Please think about submitting material to the UMAP Journal Associate Editor UMAP Journal (Joanna)

- General article on Mathematical Modeling at the undergraduate level
- Model Reality Check
- UMAP Module – exercise with solutions
- Minimodule – short exercises & answers
- On Jargon

THANK YOU!

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