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STUDENT VERSION
Ball Drop in Water

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STATEMENT

A ball of mass m (gm) and radius r (cm) is released from rest in a liquid of resistance to motion of μ (g/(cm^2 s)). The ball falls under the influence of gravity, but the liquid exerts a drag force proportional to its velocity$v$ (cm/s) and there is the force due to buoyancy which is the weight of the liquid displace by the ball $\frac{4}{3}\pi r^4 \rho \cdot g$. If y (cm) measures the vertical distance the ball has fallen, then one form of its equation of motion can be given by:

$$m \cdot y''(t) = -\pi r^2 \mu \cdot y'(t) + m \cdot g - \frac{4}{3}\pi r^4 \rho \cdot g\,. \tag{1}$$

1. Derive and defend this mathematical model using a Free Body Diagram and Newton's Second Law of Motion. The latter says that for a body of mass m its mass times its acceleration is equal to the sum of all external forces acting on the body.

   Identify all the forces in this situation and then build the differential equation which models the position beneath the surface of the ball.
2. Solve the differential equation for vertical distance (y).
3. Determine the velocity $v = y'(t)$ and speak to its attributes.
4. Suppose a copper ball of density $\mu = 8.96$ gm/cm^3 and radius$r = 0.60$ cm is dropped in water ($\rho = 1$ gm/cm^3). Determine velocity at which the ball will fall steadily after a period of time and roughly how long it takes the ball to reach this steady velocity.
5. Suppose a steel ball of density $\mu = 8.00$ gm/cm^3 is dropped in water ($\rho = 1$ gm/cm^3). Determine the radius of the ball that will fall steadily at 5.50 mph (the fastest speed of a human swimmer).