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STUDENT VERSION
School Flu Epidemic

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STATEMENT

Epidemics continue to have significant impacts on humans. The history of epidemics is well known, stemming back to the Black Death in the fourteenth century through measles, influenza, smallpox, AIDS, and the Ebola virus in more recent times. One of the most devastating epidemics was the influenza outbreak at the end of World War I. As a result of this disease, millions died around the world.

Influenza is one of the most contagious diseases spread among humans over the past several years free flu shots have now been authorized for people throughout the US. When considering this disease, we can model the population as those susceptible ($S(t)$) and those with contagious infectives ($I(t)$) over time (t). The susceptibles are those likely to catch the disease, while the contagious infectives are those infected with the disease who can communicate it to a susceptible. We can model this behavior as simple rate equations, assuming that the populations of the susceptibles and infectives are large (to neglect random differences in people). We likewise assume that those who recover are then immune.

We refer to an outbreak of influenza in an English boarding school [1].

The three differential equations that can simulate the simple behavior of an influenza epidemic (not accounting for population dynamics, i.e. natural births and deaths) can be described by the following relations:

$$\begin{aligned} \frac{dS}{dt} &= -\beta\, S\, I\,, \\ \frac{dI}{dt} &= \beta\, S\, I - \gamma\, I\,, \\ \frac{dR}{dt} &= \gamma\, I\,, \end{aligned}$$

with $S(0) = s_0$, $I(0) = i_0$, and $R(0) = 0$.

Here β is the infection rate, γ is the recovery rate (or removal rate), and $R(t)$ denotes those recovered at time t. For influenza, the infectious period is typically 1-3 days.

Let us use actual data obtained from an influenza epidemic that occurred in an English school [1]. The epidemic was started by one infective, $i_0 = 1$, and the total number of susceptibles was $s_0 = 762$ individuals. The infection rate and removal rate were estimated as $\beta = 2.18 \times 10^{-3}$ susceptibles^{-1} day^{-1} and $\gamma = 0.44$ day^{-1}, respectively.

Questions

1. How many days did it take to reach peak infection?
2. How long did it take to fully recover?
3. Notice that the number of infectives starts small and increases quickly, but then decreases gradually. Why is that? Demonstrate this fact with a plot in this situation.

REFERENCES

[1] Communicable Disease Surveillance Center. 1978. News and Notes: Influenza in a Boarding School. *British Medical Journal*. 1(6112): 586-590. `http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1603269/pdf/brmedj00115-0064.pdf`. Accessed 28 August 2018.