

## STUDENT VERSION DIALYSIS

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### STATEMENT

Kidney dialysis is a life saving procedure that is performed in out patient facilities. The patient's blood is passed through a dialysis machine and the urea in the blood is removed across a semipermeable membrane into a dialytic fluid which is very low urea or pure. The blood and the dialytic fluid need to pass by this membrane and the question in this modeling scenario is whether it is better to have the blood and the dialytic fluid flow in the same direction with the membrane separating them (*concurrent flow*) or in the opposite direction with the membrane separating them (*countercurrent flow*).

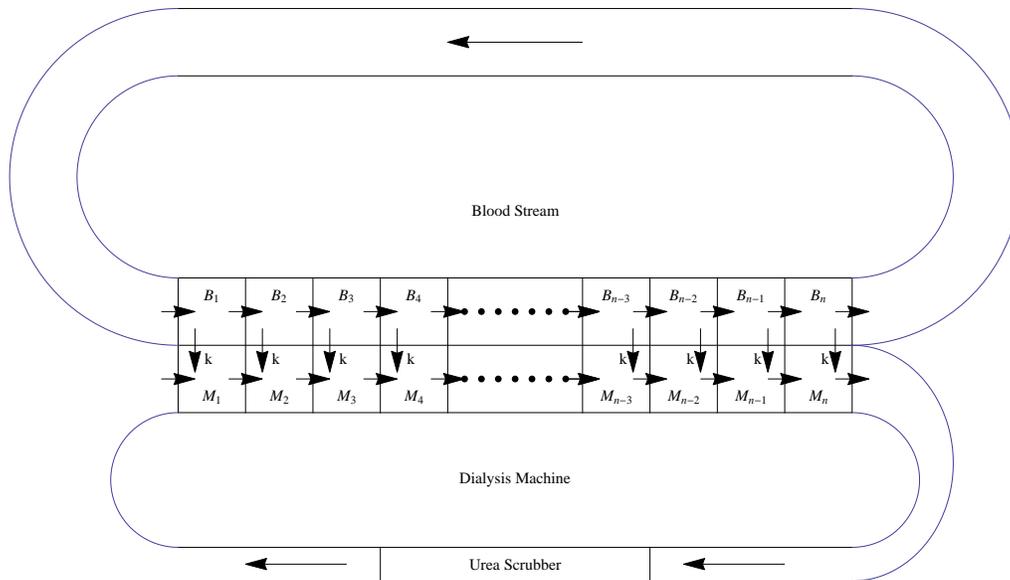
Countercurrent flow is found in nature [2]. For an excellent animation of countercurrent blood flow used to keep blood warm when it returns from cold surface exposure see [3].

Diagrams, using small compartments, illustrate flow and facilitate the modeling of concurrent flow (Figure 1) and countercurrent flow (Figure 2).

Parameters and variables in the model are identified:

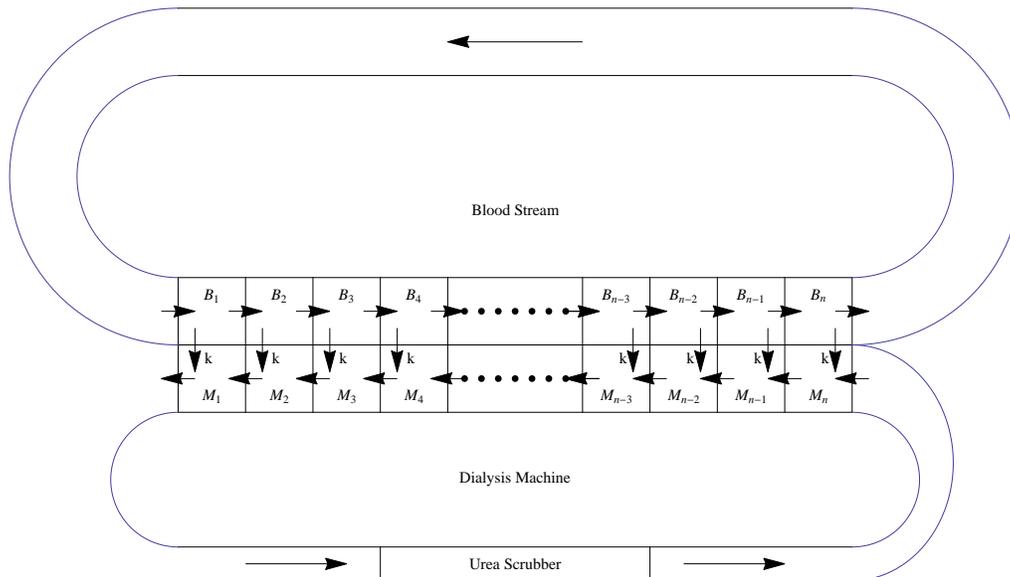
- $rB$  is the rate (L Blood/min) of flow of blood from blood compartment to blood compartment. A realistic value for  $rB$  is  $rB = 0.5$  L Blood/min. This would be difficult to change as it reflects the natural blood flow rate of the patient.
- $rM$  is the rate (L dialytic fluid/min) of flow of dialytic fluid from dialytic fluid compartment to dialytic fluid compartment. A value to start is at a flow rate slower than that of the blood flow, say  $rM = 0.1$  L dialytic fluid/min. In fact in an operator's manual [?, p. 24] the settings for "Dialysate Flow (Adjustment Range)" offerings are "300/400/500/600/700/800 mL/min." I.e.,  $rM = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8$  L dialytic fluid/min.

## Concurrent Dialysis



**Figure 1.** Model of concurrent dialysis in which the dialytic fluid flows in the same direction as the blood flows.

## Countercurrent Dialysis



**Figure 2.** Model of countercurrent dialysis in which the dialytic fluid flows in the opposite direction as the blood flows.

- $k$  is the rate constant (1/min) for flow of urea from blood compartment to dialysis compartment across one way flow membrane. This is based on a concentration gradient difference between the blood and dialysis fluid compartment concentrations.  $k = .5$  1/ min is a reasonable value for a membrane, but can be changed depending upon the type of membrane used.
- IB is the initial urea levels in the blood.  $IB = 300$  (mL of urea)/(L of blood) is a reasonably high level to warrant dialysis.
- VBB is the constant volume of blood in L in the body.
- $y_{BB}(t)$  is the amount of urea in L in the body at time  $t$  in min.
- VB is the volume of blood in the blood compartments - three compartments in all at first.
- $Y_{Bi}(t)$ ,  $i = 1, 2, 3$  is the amount of urea in L in blood compartments  $i = 1, 2, 3$ , at time  $t$  in min.
- $Y_{Mi}(t)$ ,  $i = 1, 2, 3$  is the amount of urea in L in dialytic fluid compartments  $i = 1, 2, 3$ , at time  $t$  in min.

The goal of the dialysis is to reduce the concentration of urea in the body to 50 (mL of urea)/(Liter of blood). Here are some assumptions:

- Assume the TOTAL amount of blood in the body is 5 liters. This is quite reasonable.
- For a three compartment model we shall assume each of the small compartments has volume 0.1 liter. Changing this would refine or broaden the exchange compartments in the model.
- Assume that once the cleaned blood enters the patient's blood system it thoroughly mixes with that blood, in particular, the blood that enters the dialysis unit has the same concentration of urea as that which exited the unit. This could prove to be a very seriously wrong assumption. Why? How might we change our model to avoid this? In fact, the 0.1 L of blood that enters into the approximately 5 L of blood really does not alter the concentration of urea that much so this might not even make that much different. However, this is still worthy of further modification to the simple compartment model.
- The rate at which the urea flows through the membrane from blood compartment to dialytic compartment is proportional to the difference in the concentrations in these two compartments, respectively and that rate is  $k$  1/min.
- Assume that the dialytic fluid that enters the dialysis machine is free of urea.

### Activities

1. Using concurrent flow build a three compartment model for the amount of urea present in each of the three blood compartments, the blood in the body, and each of the three dialytic machine compartments. Here is one differential equation (1) to get started. This equation is for the rate of change in the amount of urea in blood compartment 1 in which urea enters from the bloodstream, leaves to go into blood compartment 2, and moves across the membrane into compartment 1 of the dialytic machine.

$$y_{B1}'(t) = r_B \frac{y_{BB}(t)}{V_{BB}} - r_B \frac{y_{B1}(t)}{V_{B1}} - k \left( \frac{y_{B1}(t)}{V_{B1}} - \frac{y_{M1}(t)}{V_{M1}} \right). \quad (1)$$

- Using countercurrent flow build a three compartment model for the amount of urea present in each of the three blood compartments, the blood in the body, and each of the three dialytic machine compartments. Here is one differential equation (2) to get started. This equation is for the rate of change in the amount of urea in dialytic compartment 1 in which urea enters from the bloodstream, leaves to go into blood compartment 2, and moves across the membrane into compartment 1 of the dialytic machine.

$$y_{M1}'(t) = r_M \frac{y_{M2}(t)}{V_{M2}} - r_M \frac{y_{M1}(t)}{V_{M1}} + k \left( \frac{y_{B1}(t)}{V_{B1}} - \frac{y_{M1}(t)}{V_{M1}} \right). \quad (2)$$

- In both cases (concurrent and countercurrent) determine how long it takes until the urea concentration in the blood compartment reaches 50 ml/(L of blood). Based on this comparison which flow, concurrent or countercurrent, would you recommend.
- Perform the same analyses for (1) - (3) using a nine compartment model.
- Pick one of the parameters, e.g.,  $r_M$  or  $k$  (changing  $k$  might mean engineering a new membrane), and perform an analysis to see how sensitive the time until the blood compartment reaches 50 ml/(L of blood) is in terms of that new parameter.
- In considering concurrent dialysis one source concludes “the best you can expect with this sort of circuit is to halve the concentration of the solute. In the end, your patient and the dialysate fluid will both have the same concentration of urea.” [1] Defend or counter this claim based on this model. If this model does not conform to this claim how might you resolve the difference? Might the circuit spoken of NOT use clean dialytic fluids at the entry of the membrane contact portion of the device?

## REFERENCES

- [1] Deranged Physiology. 2014. Dialysis. <http://www.derangedphysiology.com/php/dialysis/6-The-benefits-of-the-counter-current-circuit-in-hemodialysis.php>. Accessed 3 April 2014.
- [2] Teboh-Ewungkema, M. I. and E. P. Salatheb. 2006. The Role of Counter-Current Exchange in Preventing Hypoxia in Skeletal Muscle. *Bulletin of Mathematical Biology*. 68: 2191-2204.
- [3] Ash, L. 2000. Counter Current Heat Exchange and Selective Vasoconstriction. Edmonton, Alberta CANADA: Department of biological Science,s University of Alberta. <http://www.biology.ualberta.ca/facilities/multimedia/uploads/zoology/counter-current.html>. Accessed 14 April 2014.