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M&M Game Revisited

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STATEMENT

It is universally assumed that the probability of an M&M chocolate, when tossed, falling with the M side up is $\frac{1}{2}$. For this reason the M&M game [1] results in the stable population of $2Q$, where Q is the number of M&M's added at each time step.

As a reminder of the game in [1] we quote the rules:

> Gently shake the M&Ms out onto the desk (you might want to use a paper plate to catch the M&Ms and keep them clean as well). We determine for each M&M if it lives or dies. If the M shows on top or up this M&M dies, otherwise there is life for this M&M. Upon death you should remove the M&M from the population (set these aside as we will need them for another experiment), count and note down the number of M&Ms who survive, and thus put fewer M&Ms back into your container for the next iteration. Do this over and over and record your results.[1]

The goal here is to find the probability distribution function (pdf) for the probability, q, that

$$Pr(\text{one randomly chosen M\&M falls with M up when tossed}).$$

How to play and collect data

Using a pack of M&M's and play the M&M game [1] many times, say 30 times and report what you believe is a stable value from these outcomes. The difference formula for this activity is given in

$$P_{n+1} = qP_n + Q \quad \text{where} \begin{cases} P_n & \text{number of M\&M's at time step } n \\ q & Pr(\text{one randomly chosen M\&M falling M up}) \\ Q & \text{number of M\&M's added at each time step} \end{cases} \tag{1}$$

The stable solution is when $P_{n+1} = P_n = P$ for large n. This results from (1) and analysis show:

$$P = qP + Q \implies P(1-q) = Q \implies P = \frac{Q}{1-q}\,.$$

Example

Suppose there are 15 students in the class and each one reports the stable number after 30 tries. With $Q = 10$ the results are given below for students S_i, $i = 1, 2, \ldots, 15$.

Student	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15
Number	22	22	23	19	19	19	24	18	19	20	21	21	22	24	18

We have

$$P_{\text{stable}} = \frac{Q}{1-q} \implies q = 1 - \frac{Q}{P_{\text{stable}}}$$

Using frequencies and the above formula we end up with the probability mass function (pmf) or probability density function (pdf) for q:

$$f(q) = \begin{cases} 2/15 & q = 8/18 \approx 0.4444 \\ 4/15 & q = 9/19 \approx 0.4737 \\ 1/15 & q = 10/20 = 0.5 \\ 2/15 & q = 11/21 \approx 0.5238 \\ 3/15 & q = 12/22 \approx 0.5455 \\ 1/15 & q = 13/23 \approx 0.5652 \\ 2/15 & q = 14/24 \approx 0.5833 \end{cases}$$

Using this we can find the probability of a random M&M falling with M up in the following way:

$$\begin{aligned} q &= Pr(\text{one randomly chosen M\&M falling M up}) \\ &= \sum_{k=18}^{24} Pr(\text{one randomly chosen M\&M falling M up} \mid \text{q = 1 - 10/k }) \cdot Pr(q = 1 - 10/k) \\ &= \left(\frac{8}{18}\right)\left(\frac{2}{15}\right) + \left(\frac{9}{19}\right)\left(\frac{4}{15}\right) + \left(\frac{10}{20}\right)\left(\frac{1}{15}\right) + \left(\frac{11}{21}\right)\left(\frac{2}{15}\right) + \left(\frac{12}{22}\right)\left(\frac{3}{15}\right) \\ &\quad + \left(\frac{13}{23}\right)\left(\frac{1}{15}\right) + \left(\frac{14}{24}\right)\left(\frac{2}{15}\right) = 0.5133\,. \end{aligned}$$

ACTIVITIES

Activity 1

Derive (1) with explanation.

Activity 2

Conduct your own experiment with classmates leading to your estimate of q, the probability of a random M&M falling with M up. From your results do you thin it is a reasonable assumption that the probability of a random M&M falls with M up is $\frac{1}{2}$?

REFERENCES

[1] Winkel, Brian. 2015. 1-001-S-MandMDeathAndImmigration. `https://www.simiode.org/resources/132`. Accessed 27 Auogust 2018.