Flipping the Classroom
Joint Mathematics Meetings – Baltimore
9:00 AM, Saturday, 18 January 2014, Room 337, BCC

Sylvanus Thayer Flipped Out
Method Ahead of Its Time

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Above . . . . view of Hudson River from Brian Winkel's office in Thayer Hall at United States Military Academy. There are no office Windows in Thayer Hall!

Next slide . . . . view from Brian Winkel's office window at United States Air Force Academy. This shows the true meaning of **Distinguished** Visiting Professor.


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Abstract

Sylvanus Thayer is dead, but the **Thayer Method of teaching – modified as with all things – lives on at West Point**. We discuss how **students come to class prepared from reading and attempted scaffolded activities before class and how that energizes the classroom activities and learning.** We show how such a method works in a mathematical modeling course in which students are to prepare some initial inquiries into a modeling scenario before class and then develop a more sophisticated and solid model through classroom interactions under the guidance of a classroom teacher.

In 1815, Sylvanus Thayer (1785-1872) and William McRee were sent to Europe to study the continental military system and schools. Most of their time was spent at the École Polytechnique. Their other mission was to obtain texts. Thayer and Ray brought back over a thousand books, as well as maps and pamphlets from France. Most of these now reside in the Thayer Collection of the Academy Library.

Upon returning to the States in 1817, Thayer was made the third Superintendent of the Academy. **He went on to be known as the Father of the Academy** for the work he did to **revise and revitalize the Academy and its curriculum**. However, it must be said that **many of his changes were started or advocated by his predecessors. He modeled many of his changes, both academic and military, on the École.** [Rickey, Shell-Gellasch, 2010.]

Sylvanus Thayer

Father of the United States Military Academy

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would just flip over modern day teaching methods in which students prepared before class and worked problems in class.

In fact, he did more than flip he made it THE teaching approach at the United States Military Academy over 200 years ago and it is still used there today.

We call it the Thayer Method!

“He also created a teaching method known today as the Thayer Method, which emphasizes self study and daily homework, as well as small class size.” [Wikipedia]

The Thayer Method in its purest form consisted of cadets reading and doing mathematics “at home” in the barracks the night before class and then coming to class to recite their problems at the blackboard in front of instructor and classmates.

Cadets were graded on their board work and presentation and at the end of each time period (day or week) were reseated in each class – top to bottom, and then throughout all sections (after resectioning) - again, top to bottom.

High accountability and discipline – not surprising it is the Army!

Then no collaboration was permitted – Honor Code violation.
Now collaboration and cooperation is expected.

The method is used in some form today at USMA and USAFA.

Usually the previous class ends with a look ahead presented by the teacher.

The students already have the syllabus, usually on-line (now!)

Suggested problems from the reading (formerly called “Drill Problems”) are listed and go from easy one-step to complex.

Cadets are admonished to do the reading in the text and to try the problems as far as they can go. At the Academies cadets are paid to go to school and so it is their duty to do homework!

At USAFA one problem is recommended and often cadets place that problem individually on the board.

This posting by all and then briefing by selected cadets serves as a guide for grading at USAFA. On exams (often no homework is collected, but there is a project or two in each class) a 0-5 point grading system is used on mathematical content and communication.

In 2009-2010 the grading rubric was wonderful as it simply asked if the problem submission was Outstanding, Good, Average, Deficient, or Failing with emphasis on Well-Executed, Well-communicated, and Essentially Correct. Consensus is easily reached by faculty in trial runs and comparisons from a random section and there is no squabbling about points. Problems are then weighted and multiplied by the 0-5 score and then tallied.

So how did I flip the classroom in a modeling course?

It was really a flip-flop – with all due apologies to politicians.

By flip-flop we mean there was more than just read and prepare the night before.

Let me explain.

Here is what happened at its best . . . . . .

During class we would introduce a modeling scenario in one of several ways.

(1a) Class serves as a consultant to visiting client (either teacher in “disguise” or real client), e.g., ecologist studying rattlesnake population modeling on West Point reservation.

(1b) Vague situation introduced by teacher with not much detail, but an issue or looming question for thought. Overnight thinking, tinkering, and mulling on the part of students.

(3) Next day’s class we would work on the scenario, perhaps teacher serving as source of data or Internet accessed.

(4) For next class or say two classes down the road, students submit a write up with good communication and analysis.

We always introduced the mathematics with a modeling situation in which addressing a modeling situation was paramount and the mathematics followed and was therefore highly motivated.

So we never had students read mathematics ahead; only thinking about a modeling scenario overnight for class.

Here are examples of the kind of modeling we could lead with on the day before and ask students to think about in preparation for class on the next day.

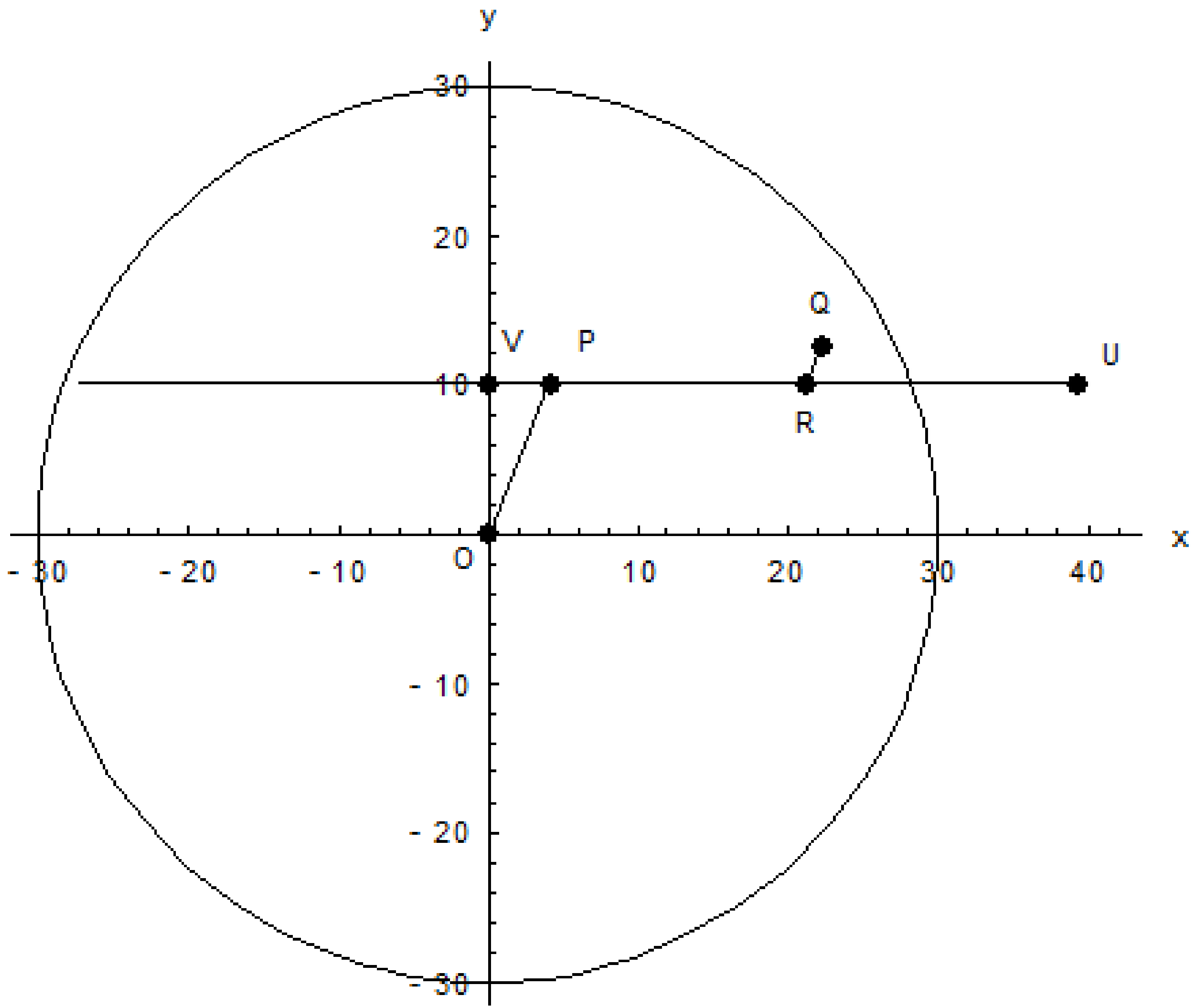
We then would work on their ideas in class.

Getting to the Mall – Getting Malled

The average driving speed to reach a shopping mall in a suburban area through unimproved roads is 30 miles/hour. People seem to be willing to spend no more than one hour of driving time to reach the mall. Hence the "neighborhood" of the mall determined by this transportation constraint is a circular region centered at the mall and having a 30 mile radius

Suppose a new east-west highway is built, passing ten miles due north of the shopping mall and that the driving speed on the highway is 55 miles/hour.

Determine the new "neighborhood" for the shopping mall in view of the option to take this east-west highway.



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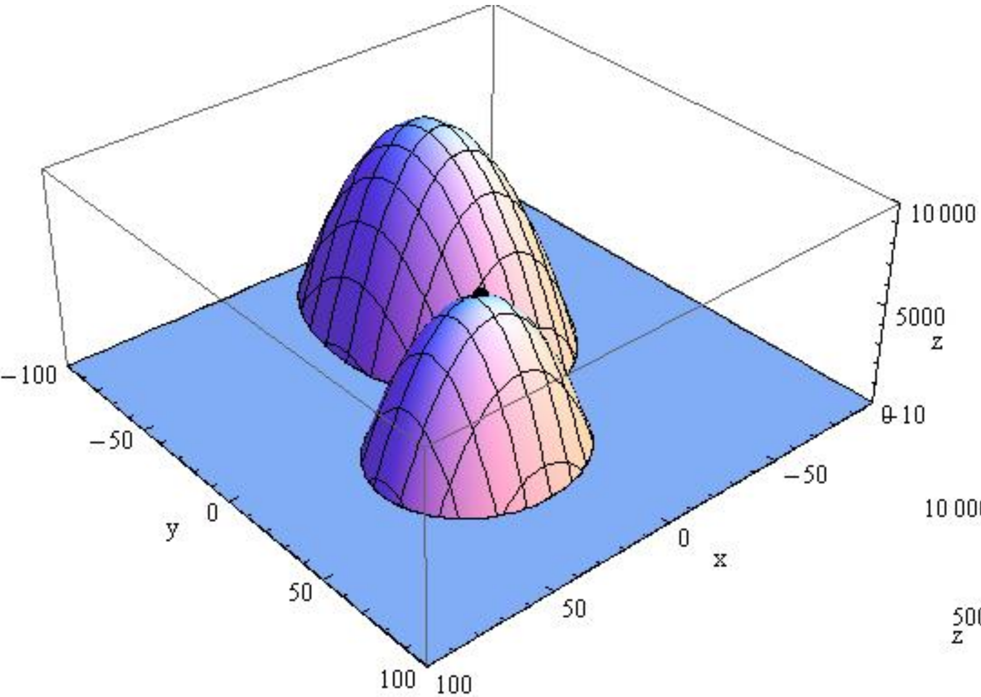
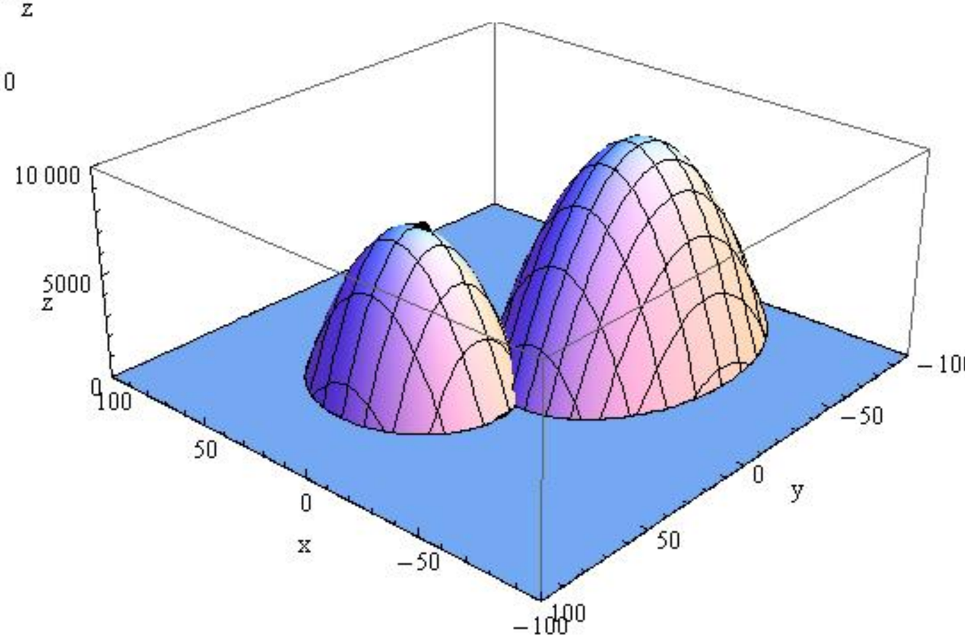
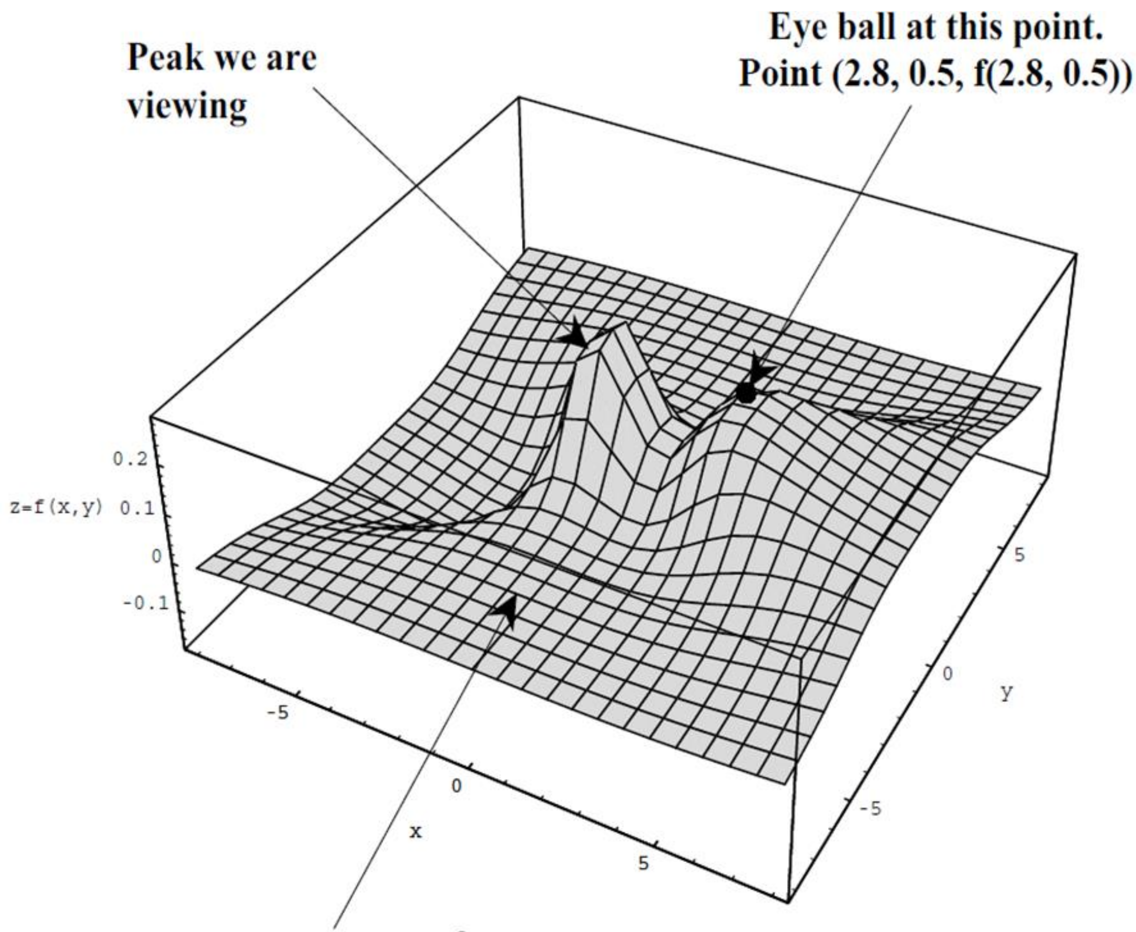


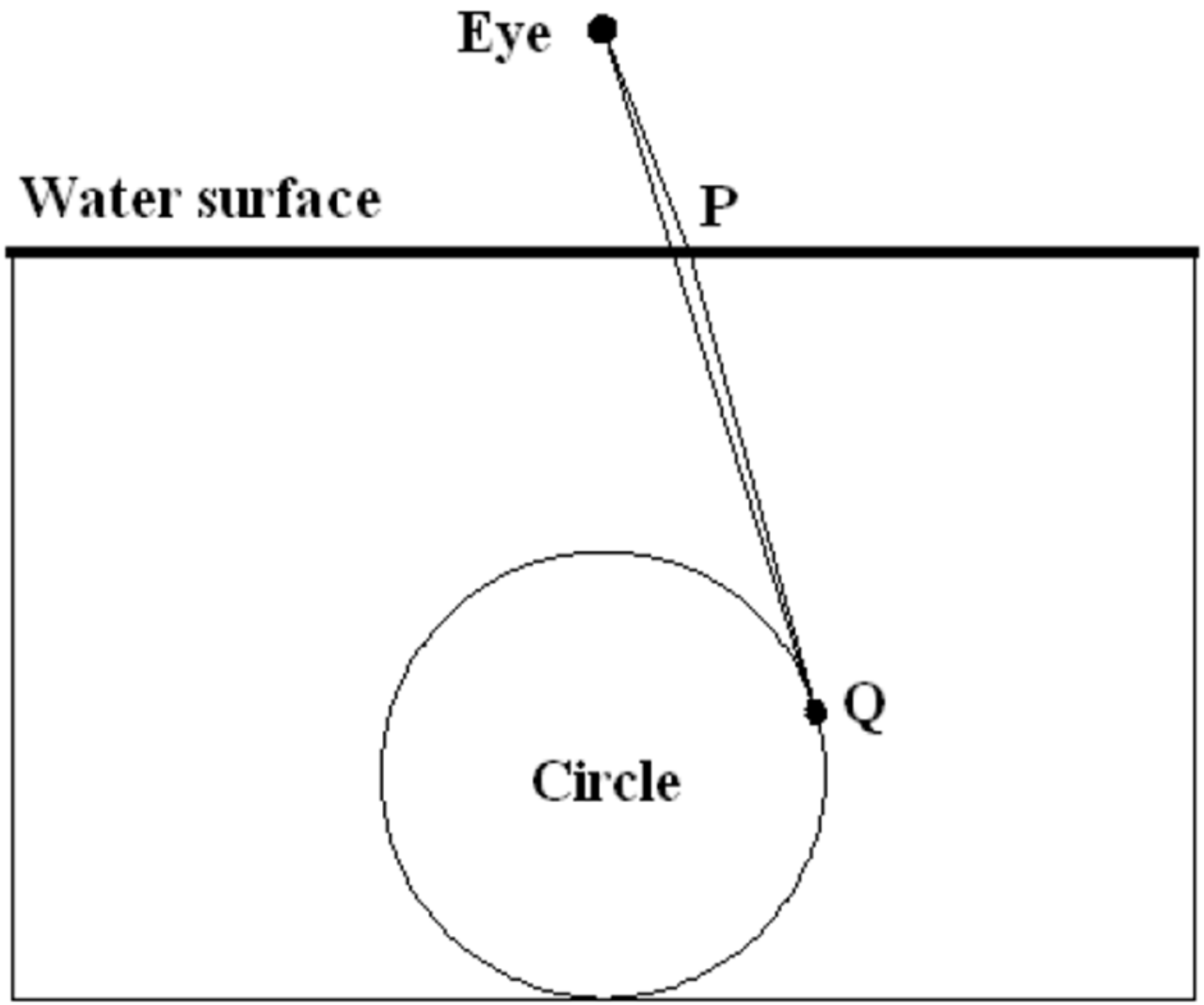


Put your eye on the dot on the slope of the mountain in the foreground and ask what you can see on the mountainside in the background. Two views presented.

Go home and think about it and come back to class prepared to work on this.



$$\text{Surface } z = f(x,y) = \frac{(x^3 - 3x + 4)}{(x^4 + 5y^4 + 20)}$$



Can we see more of the surface area of a ball at a given distance if we submerge it in water? Assigned with no picture.

In this case consider how good our model was in comparison to some previously taken data or ACTUALLY taken that day in class.

Model the sublimation of a block of carbon dioxide.

That's it!

What happened in class the next day?

In response to the prompt, “Model the sublimation of a block of carbon dioxide.” students come to class with a number of questions, assumptions, misconceptions, and models.

We ask them to write up some of their ideas – ones they really believe in and ones they have doubts about – at the board.

We go around the room and discuss the questions, the issues, the merits, the counterexamples or obvious reality checks on their assumptions or models.

We spend time in small groups discussion how their models might be solved and verified if we had some data.

Aha! We provide them with data so they can work on building and validating their model during class and for the next class where we wrap up our inquiry and require a write-up.

What kinds of things do they do, do they assume?

What models do they offer when out on their own?

It depends on whether this is a first modeling activity or one that is well in to the course.

Some just jump in and suggest exponential decay as a model and they need the half-life for the mass of dry ice as it sublimates.

Others suggest this begins with the assumption that the rate of sublimation (loss of mass) of carbon dioxide depends upon how much mass there is, i.e. their model is

$$m'(t) = -k\, m(t) \quad \text{with} \quad m(0) = m_0.$$

Usually there is a science or engineering major or two in the class and this model $m'(t) = - k\, m(t)$ with $m(0) = m_0$ assumes that the rate of sublimation or change in mass is merely dependent upon the mass present.

If sublimation is the release of carbon dioxide from the solid state to the gaseous state perhaps rate of sublimation or change in mass is proportional to surface area.

This leads to a model like this:

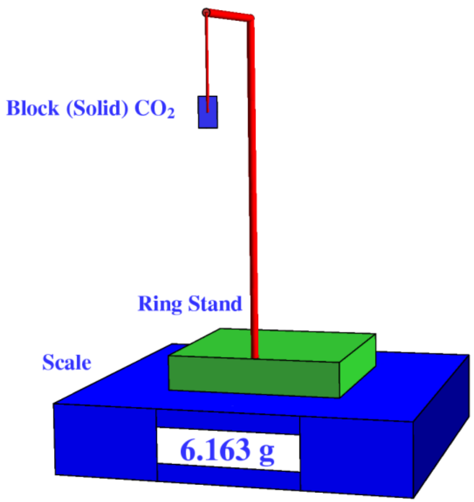
$$m'(t) = - k_1\, m(t)^{(2/3)} \qquad \text{with} \qquad m(0) = m_0$$

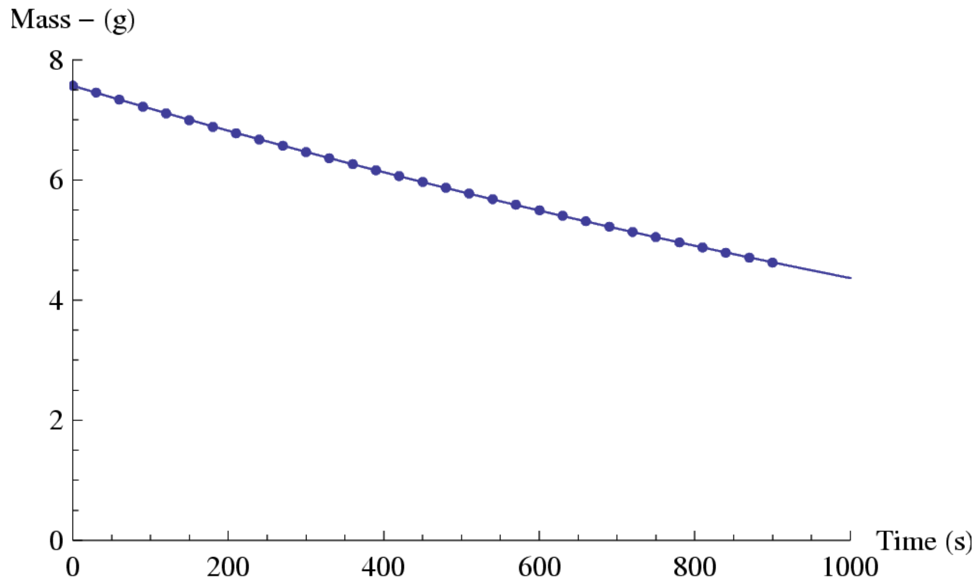
for mass is directly proportional to volume. What assumption here – constant mass density? Then units of surface area are proportional to units of volume cube rooted and squared, i.e. to the 2/3 power.

In class after their trials we come to consider a model like this:

$$m'(t) = - k_1 \, m(t)^r \qquad \text{with} \qquad m(0) = m_0$$

where r “should be” 2/3.





What we find with several data sets collected is

$$r = 0.7512 \qquad \text{and} \qquad k_1 = 0.0008506 \,.$$

This leads to more discussion as to what this r might mean?

After exponential growth, after traditional predator prey – each developed after pre-class examination and reflection – we examine the usual stability issues of predator prey model and ask if this model is realistic.

$$N'(t) = a\,N(t) - b\,N(t)\,P(t) \quad , \quad P'(t) = -\,c\,P(t) + d\,N(t)\,P(t)\,.$$

We devote lots of class time for discussion on the meaning of the coefficients a, b, c, and d and whether or not they are realistic.

At the end of class we ask if the term $-\,b\,N(t)\,P(t)$ is realistic, e.g, supposing N is prey (pizza) and P is predator (us). Now if $P = 1$ then $-b\,N(t)\,P(t) = -\,b\,N(t)*1 = -\,b\,N(t)$ and this says one person ($P = 1$) can consume the same percent (b) of whatever number of N passes before them per unit of time. Make sense?

Students are sent home to attempt to address this weakness in the predator-prey model and return to class with suggestions.

We might spend half the next class attempting to model saturation in predation finally coming up with a reasonable term

$$N'(t) = a\,N(t) - b\,\frac{N(t)}{K+N(t)}\,P(t), \quad P'(t) = -\,c\,P(t) + d\,\frac{N(t)}{K+N(t)}P(t)\,.$$

Then we return to equilibrium and stability issues and other possible changes in the model, e.g., logistic growth for N(t) instead of exponential growth. Again, overnight preparation and thinking.

We build stochastic process models from reasonable axioms based on some common sense notions and then get into queuing theory as an example.

We discuss simple notions like average size of queue, average time until service, change in service and arrival rates, etc.

Then we discuss the distinct notions of reneging (i.e. stepping out of line) and balking (i.e. not entering the line if it is large).

We challenge them to model these phenomena on their own and when we return to class we discuss their efforts.

Thus flipping the class means expecting students to invest in their learning in class before they come to class and not just after class with homeworks.

We expect them to invest before class AND during AND after!

Furthermore, in a modeling class or applied setting we expect students to build terms in a model and to ask "what if" type questions before coming to class and be prepared to do so with regard to other posings by classmates.

Coming to class having invested makes a big payoff in learning and in participation in class.

Sylvanus Thayer knew that and we still practice it at USMA
. . . . and elsewhere!

References

Rickey, V. F. and A. Shell-Gellasch. 2010. Mathematics Education at West Point: The First Hundred Years. http://www.maa.org/publications/periodicals/convergence/mathematics-education-at-west-point-the-first-hundred-years-introduction. Accessed 30 October 2013.

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Thank you for engaging.