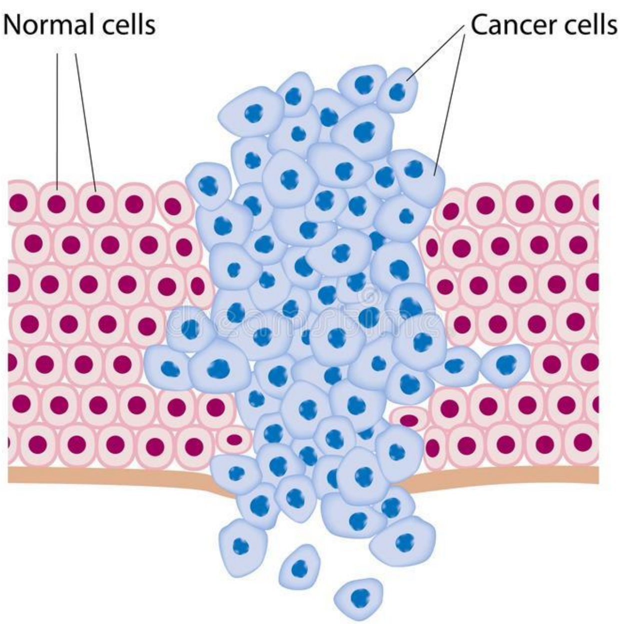
UNITED STATES MILITARY ACADEMY
WEST POINT

MA153 Project 1 Intro

Comparing Tumor Growth Models



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Agenda

1. Calendar
2. Motivation
   – Intro Video
   – Tumor Growth Diagram
3. Multi-Tool Reminder
4. Template
5. Rubric

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Calendar Project 1 Timeline

11-Sep **PSL 3** **1**	**12-Sep** **LSN 12** **2**	**13-Sep** **1**	**14-Sep** **LSN 13** **2**	**15-Sep** **1**	**A/D**
WPR 1 Review	**WPR 1**	**No Class Meeting** *Modified Schedule*	Project 1 Intro **Tech Lab 1 DUE**	**No Class Meeting**	**BEAT OHIO STATE (away)**
18-Sep **LSN 14** **2**	**19-Sep** **1**	**20-Sep** **2**	**21-Sep** **LSN 15** **1**	**22-Sep** **LSN 16** **2**	**A/C**
Project 1	**No Class Meeting**	**No Class Meeting**	Preliminary Theory IVP and BVP (4.1.1) Homogeneous (4.1.2) **Project 1 DUE**	Reduction of Order (4.2)	**BEAT TULANE (away)**
25-Sep **LSN 17** **1**	**26-Sep** **LSN 18** **2**	**27-Sep** **1**	**28-Sep** **LSN 19** **2**	**29-Sep** **LSN 20** **1**	**F**
Homogeneous Equations with Constant Coefficients (4.3) *Distinct Real & Repeated Real*	Homogeneous Equations with Constant Coefficients (4.3) *Complex Roots*	**No Class Meeting**	Nonhomogeneous (4.1.3) Undetermined Coefficients (4.4)	Tech Lab 2 (Second Order DEs)	**BEAT UTEP**

1. **Introduced (14 Sep)**
2. **IPR Plan (18-19 Sep)**
3. **Due Start of Class (21 Sep)**

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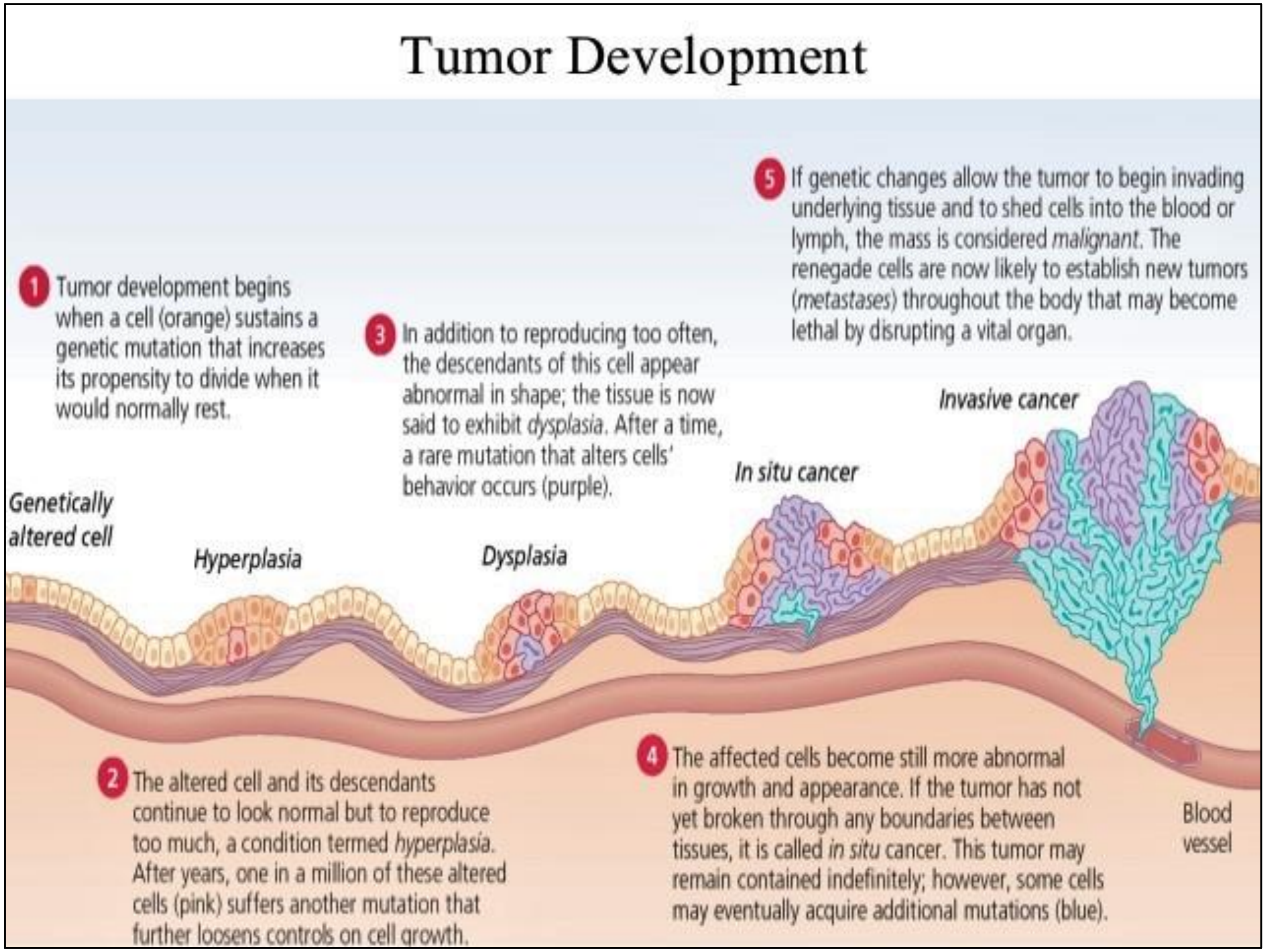
Motivation

Intro Video: [Dr. Komaroff, Harvard Med School]



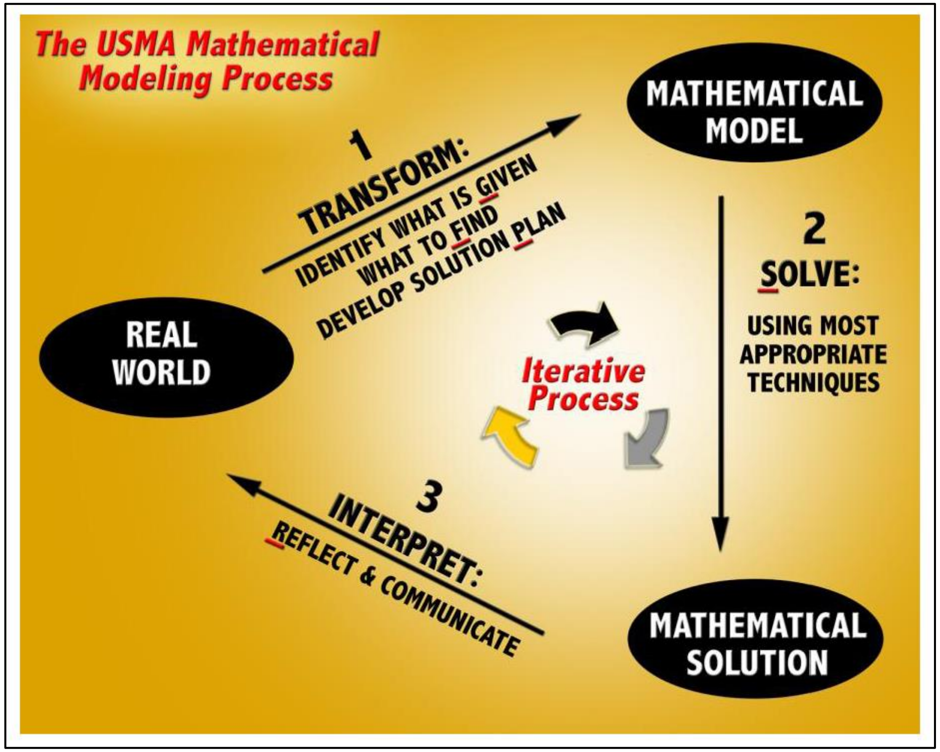
Model Development

- Transform experiment into model using
  - Known Laws of Nature
  - Observations
  - Logic
- Solving DEs
  - Requires classifying DEs
  - Different Approaches
    - Graphical (Qualitative)
    - Analytic
    - Numerical

    (Quantitative)
- Interpret
  - Develops Model
  - Initiates Iterative Process

    Model Prototype: Simple;

    Iterative Step: Prototype + Complexity



MA153 ODE LEATHERMAN

Analytic (Exact)

Separation of Variables

§9.3 Separation of Variables

$$\frac{dy}{dx} = g(x)f(y) \;\rightarrow \int \frac{1}{f(y)}dy = \int g(x)dx$$

$$\frac{dy}{dx} = \frac{g(x)}{f(y)} \quad \rightarrow \int f(y)dy = \int g(x)dx$$

- 1^{st} Order
- Separable

Integrating Factor

§9.5 Integrating Factor

$$\frac{dy}{dx} + P(x)y = Q(x)\,; \quad I(x) = e^{\int P(x)\,dx}$$

$$y(x) = \frac{1}{I(x)}\left[\int I(x)\,Q(x)\,dx + C\right]$$

- 1^{st} Order
- Linear

Characteristic Equation

§17.1 2^{nd} Order Linear, Homogeneous ODEs with Constant Coefficients

$ay'' + by' + cy = 0,\; y(x_0) = y_0,$ and $y'(x_0) = y_1$

Characteristic (Auxiliary) Equation: $ar^2 + br + c = 0$

- 2^{nd} Order
- Linear
- Constant Coefficients

Systems of ODEs

$$\frac{dx_1}{dt} = 3x_1 - 4x_2$$

$$\frac{dx_2}{dt} = 4x_1 - 7x_2$$

Solution: $x_1 = e^{-5t}, x_2 = 2e^{-5t}$

- 1^{st} Order
- Solve for Equilibrium Points
- Verify a Solution

Graphical

VectorPlot with Tech

Slope Field: y vs. t (dep v. ind)
Phase Portrait: y vs. x (dep 1 v. dep 2)

- Critical Points
- Explore Long Term Behavior

Numerical

Euler's Method

§9.2 Euler's Method

$$\frac{dy}{dx} = f(x, y),\; h = \Delta x, \text{ and } n = 0, 1, 2, 3$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + h \cdot f(x_n, y_n)$$

- Approximate a solution

Power Series

Assume a Power Series

§17.4 Power Series Solution to ODEs

$$y = f(x) = \sum_{n=0}^{\infty} c_n x^n$$

$$y' = f'(x) = \sum_{n=0}^{\infty} (n+1)c_{n+1}x^n$$

$$y'' = f''(x) = \sum_{n=0}^{\infty} (n+1)\,(n+2)c_{n+2}x^n$$

- 2^{nd} Order
- Linear
- Variable Coefficients

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Questions?