SIAM ED18 Minisymposium
Portland OR USA
9-11 July 2018

Modeling in Differential Equations Courses – SIMIODE Resources and Community

Brian Winkel, Director SIMIODE, Cornwall NY USA

www.simiode.org Director@simiode.org

Four 25 Minute Sessions in this Minisymposium

Brian Winkel, SIMIODE Cornwall NY
Title: **SIMIODE Community and Purpose –
Supporting Modeling in Teaching Differential Equations**

Corban Harwood, George Fox University, Newberg OR
Title: **Engaging in Pedagogical Development, Applied Scholarship, and
Professional Service with SIMIODE**

Barbara Edwards, Oregon State University, Corvallis OR and
Jennifer Czocher, Texas State University, San Marcos TX
Title: **Assessment/Evaluation Plans and Results on SIMIODE Programs –
Workshops and SCUDEM - Student Competition**

Corban Harwood, George Fox University, Newberg OR with Alexandra Hanson,
Laura Nosler, and Philip Nosler, Clackamas Community, Oregon City OR
Title: **SCUDEM 2018 - Student Competition Using Differential Equations Modeling
Organization, Activities, Participation**

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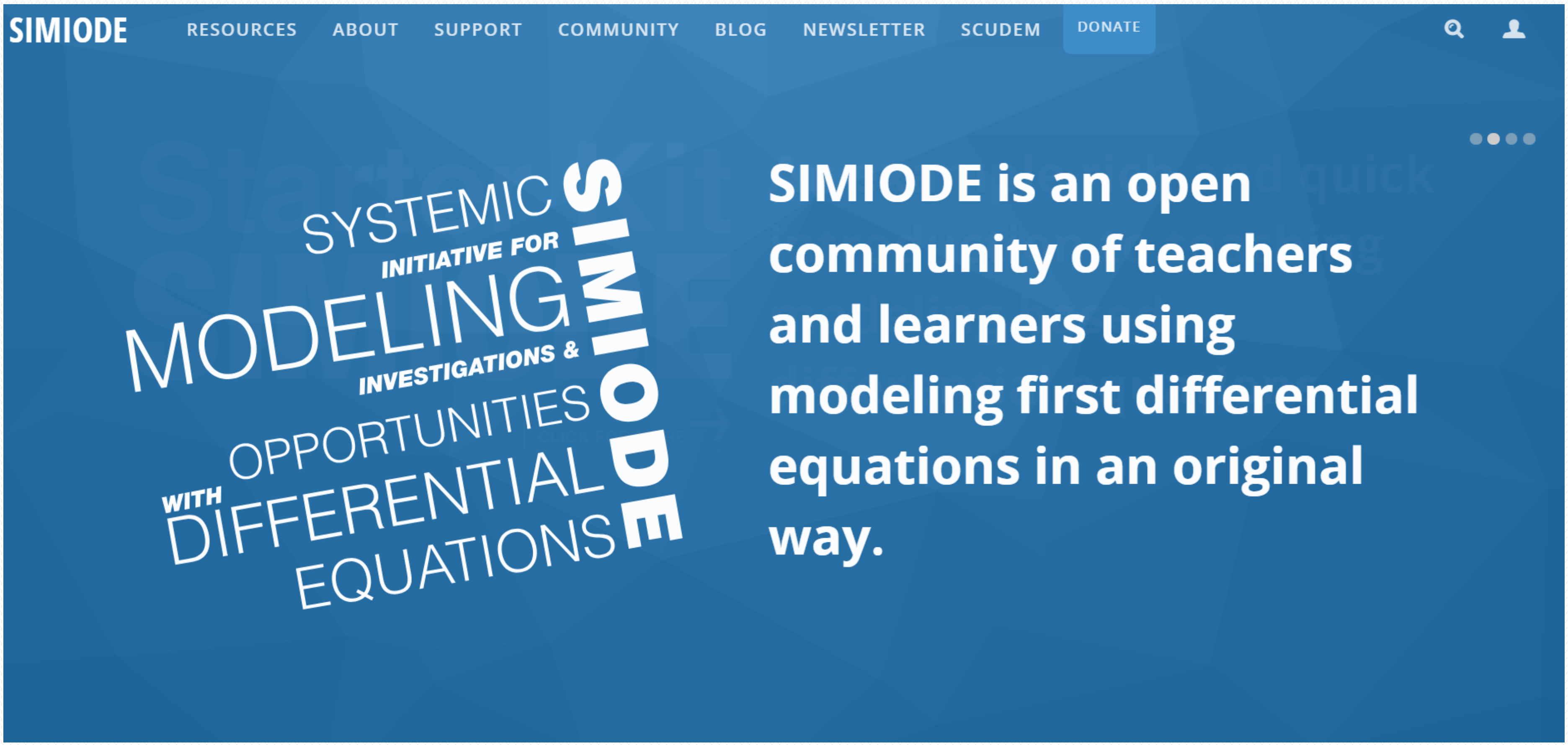
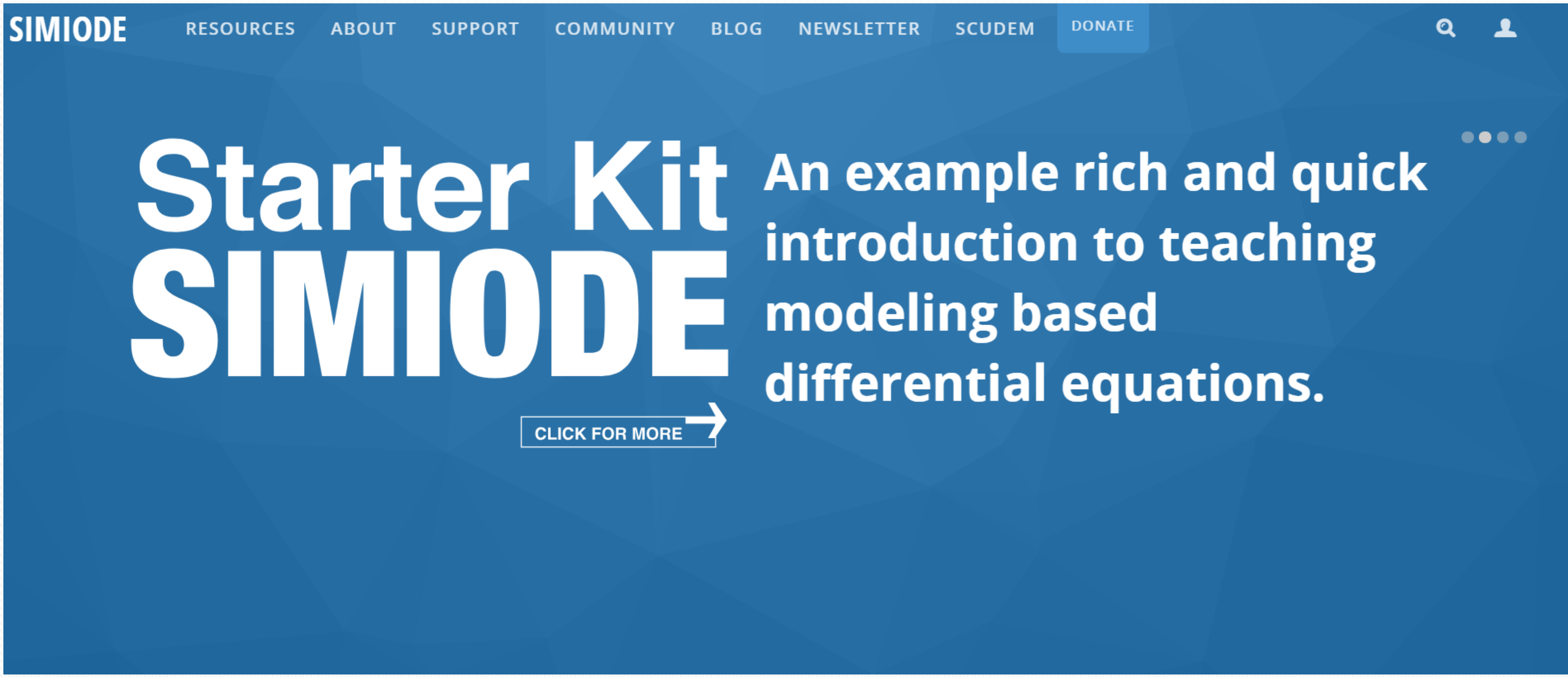


SIMIODE has been using the HUBzero platform developed by Purdue University since 2013.

We welcome a new member to the HUBzero platform family today!

All is FREE at SIMIODE!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SIMIODE
An example rich and quick introduction to teaching modeling based differential equations.
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Starter Kit

Quick Start

MATERIALS FOR TEACHING MODELING WITH DIFFERENTIAL EQUATIONS

SIMIODE offers class materials and support for faculty who want to use modeling to motivate and teach differential equations. Everything in **SIMIODE** is FREE and all materials are offered according to a Creative Commons license.

An example of a first day project M&M Simulation of Death and Immigration will give you a good idea what to expect from a modeling scenario. This scenario has been used by thousands of students over many years as a first day activity introducing differential equation. After you have looked it over you might take a look at these other choices.

First Day Activities

Ant Tunnel Building, a simple approach to create differential equation model

Modeling Spread of Oil Slick, poor data for a differential equation model

Modeling Scenarios for Early Topics

Falling Column of Water illustrates Torricelli's Law

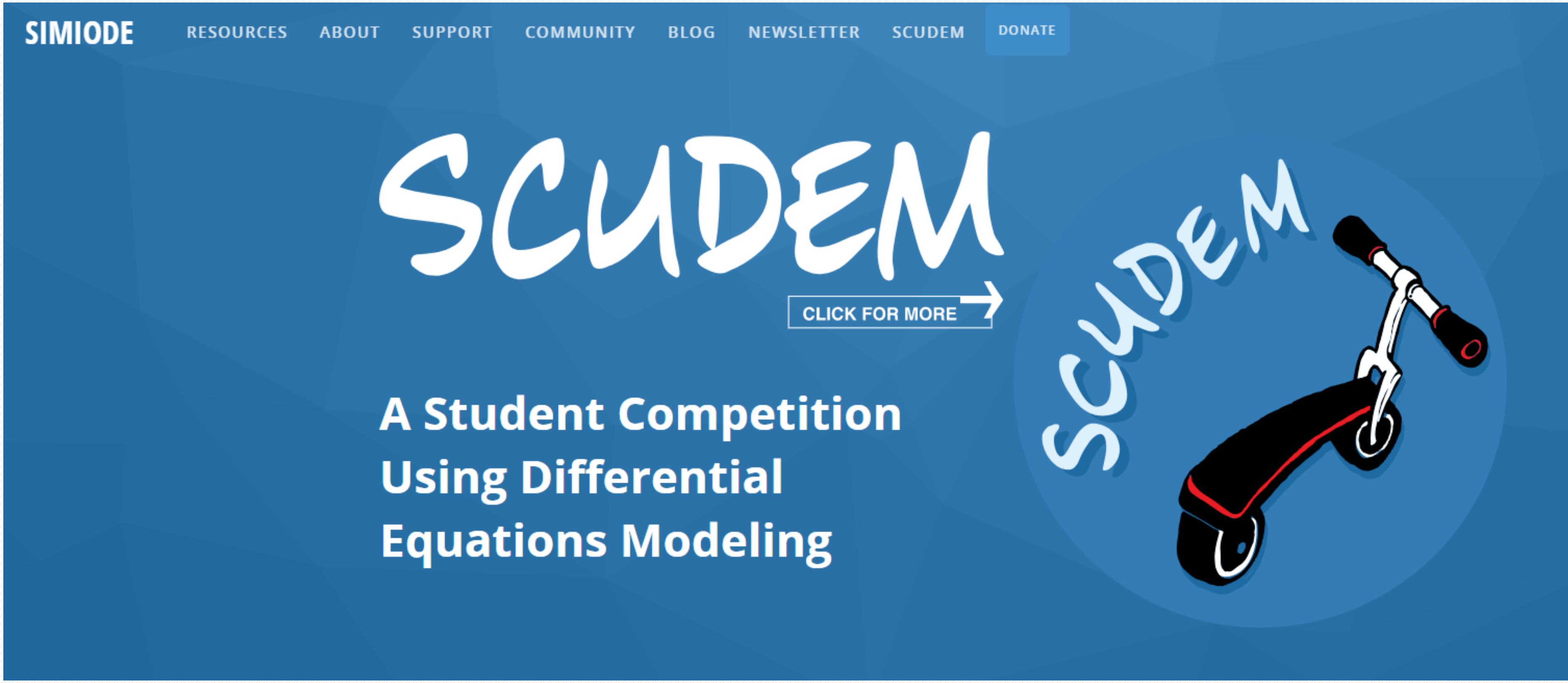
Chemical Kinetics based on students' chemistry texts

Spread of the word JUMBO in literature exploring exponential growth

Dissipation of retinal gas bubbles following surgery using clinical data

Adsorption simulation on a wall with first-order differential equation

Others scenario samples include Potato cooling; Finance of savings and loans; student simulation of spread of common cold.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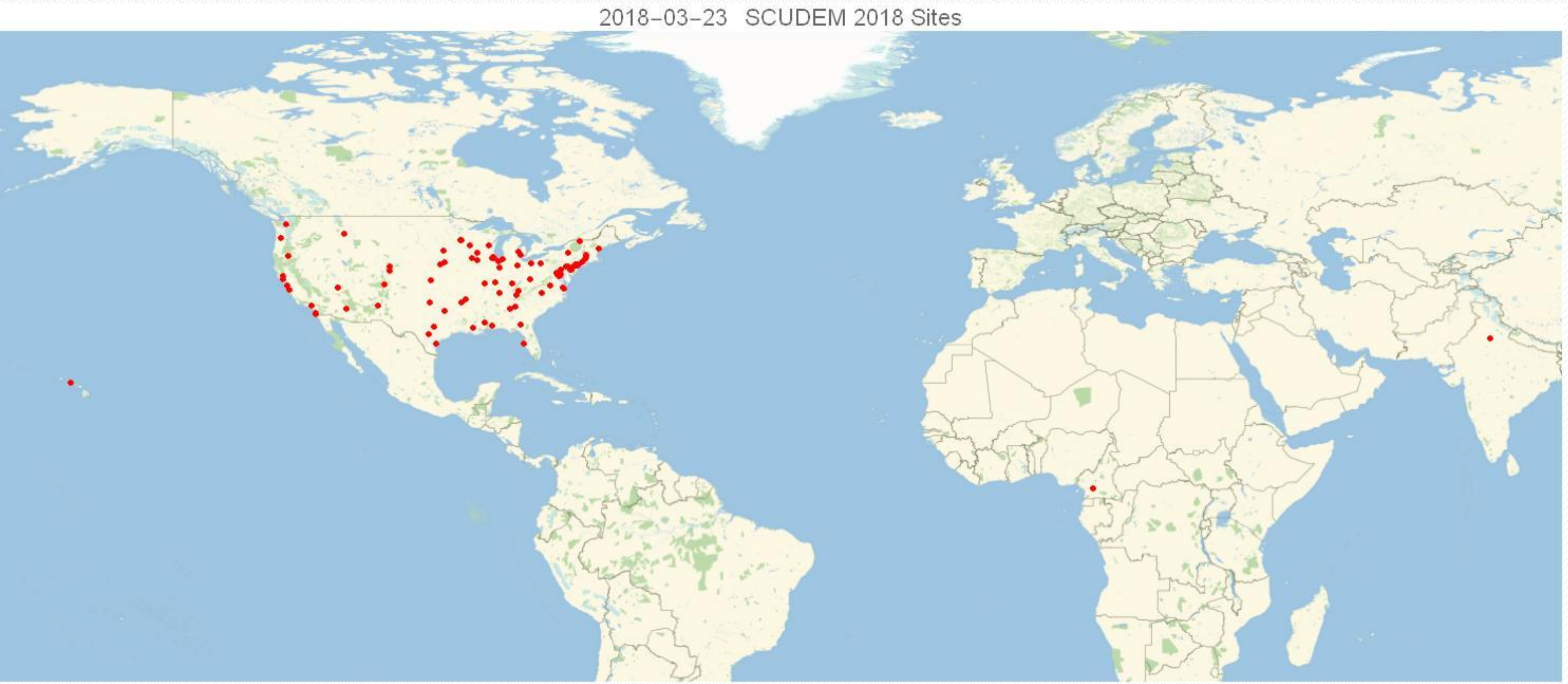
A Student Competition
Using Differential
Equations Modeling
SCUDEM

SCUDEM II 2018, 21 April 2018 - 400+ students, 130 teams and coaches, 40 sites



SCUDEM III 2018, 27 October 2018
Sign up to be host site, registration opens 1 August 2018

Articles and Publications

These are materials for which you have permission to reproduce.

Browse ›

Competitions-SCUDEM

Everything about SIMIODE's Student Competition Using Differential Equations Modeling.

Browse ›

Free Online Texts

Find annotated descriptions and links to Free Online Texts.

Browse ›

General Resources

These are broad sets of resources for teaching and include pointers to on-line class notes, presentations, videos, blogs, good questions, etc.

Browse ›

Modeling Scenarios

These are key pedagogical components of SIMIODE in which a modeling situation, rich in detail, motivates the study of differential equations. These are peer reviewed.

Browse ›

Potential Scenario Ideas

These are materials, thoughts, pointers, summaries, etc. about your modeling scenario ideas. As much detail as possible should be offered. These are not complete and may be a solicitation for collaborators.

Browse ›

Presentations

These are singular materials (usually one set of notes or one PowerPoint, video, outline, etc.) which you believe SIMIODE members will value for which you have permission to reproduce.

Browse ›

Sample Syllabi and Course Reflections

Syllabi and course experiences and narratives which illustrate SIMIODE approaches. Please examine and reproduce for your own classes.

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Technique Narratives

These provide techniques and strategies for solving differential equations and must be clear, with examples and activities or exercises for students. These are peer reviewed.

Browse ›

Resources: Modeling Scenarios

Type: Modeling Scenarios ▼ Go

Tag	Resources (Sort by Title ▼)
[All]	1-001-S-MandMDeathAndImmigration
absorption (2)	1-001-T-MandMDeathAndImmigration
acceleration (1)	1-001A-S-MandMDeathImmigration
Acorns (2)	1-001A-T-MandMDeathImmigration
administer (2)	1-001B-S-MAndM-DeathImmigrationMystery
administration (2)	1-001B-T-MAndM-DeathImmigrationMystery
air conditioning (2)	1-001pgf-S-BirthDeathImmigration
air friction (1)	1-001pgf-T-BirthDeathImmigration
air management (1)	1-002-S-Tossing
air resistance (2)	1-002-T-Tossing
airshed (2)	1-003-S-CollegeSavings
Akaike Information Criterion (4)	1-003-T-CollegeSavings
algae (2)	1-004-S-MicroorganismImmigration
amplitude (2)	1-004-T-MicroorganismImmigration

Search

logistic

Search

RESULTS (PAGE 1 OF 4)

logistic | logistic growth | logistic equation | logistic function | logistic model | logistic differential equation | logistic population growth | logistic differential equations | show more...

Logistic Differential Equation

Potential Scenario Ideas 10 Sep 2017 Contributor(s): Brian Winkel

Logistic Differential Equation.... 7 pp.   There is rich development of the logistic model with several extended activities and the author(s) use Maple to illustrate many points....   Keywords: differential equation, model, logistic, Maple

https://www.simiode.org/resources/4033

1997Shulman Using Original Sources to Teach the Logistic Equation

Potential Scenario Ideas 20 Jun 2015

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resources:modeling_scenarios:logistic

RESULTS (PAGE 1 OF 1)

logistic | logistic growth | logistic equation | logistic function | logistic model | logistic differential equation | logistic population growth | logistic differential equations | show more...

1-066-T-USCensusModeling

Modeling Scenarios 16 Sep 2017 Contributor(s): Jean Marie Linhart

The United States Census, conducted every 10 years, gives data on the United States population, that can be modeled with the exponential, logistic, or Gompertz functions.... Parameters in the exponential and logistic models can be estimated from per-unit population growth calculated from the data.

https://www.simiode.org/resources/4211

1-038-T-Ebola

Modeling Scenarios 28 Aug 2016 Contributor(s): Lisa Driskell

Students will use data published by the World Health Organization to model the 2014 outbreak of the Ebola virus in West Africa. We begin with a simple exponential growth model and move through the modeling process to the logistic growth model.... Students will investigate properties of the logistic growth model and will compare...

https://www.simiode.org/resources/2722

Potential Scenarios

2001-Tsoularis-Analysis of logistic growth models

Potential Scenario Ideas 21 Jun 2015

Article Review and Annotation Tsoularis, A. 2001. Analysis of logistic growth models, Research Letters in the Information and Mathematical Sciences.... Full article available at http://mro.massey.ac.nz/handle/10179/4341 . Article Abstract: A variety of growth curves have been ...

https://www.simiode.org/resources/934

2014Danielsen Sørensen Using authentic sources in teaching logistic growth: A narrative design perspective.

Potential Scenario Ideas 22 Jun 2015

Danielsen, Kristian and Henrik Kragh Sørensen. 2014. Using authentic sources in teaching logistic growth: A narrative design perspective.... henrikkragh.dk/logistisk-vaekst/ESU-handouts.... Accessed on 31 May 2015. This 12 page offering develops issues and questions about using historical materials...

https://www.simiode.org/resources/907

1986-CookeWitten- One-dimensional linear and logistic harvest models

Potential Scenario Ideas 12 Sep 2017 Contributor(s): Brian Winkel

Cooke, Kenneth L. and Matthew Witten. 1986. One-dimensional linear and logistic harvest models.... 7: 301-340. Abstract: Some of the results in the literature on simple one-dimensional, density dependent, discrete and continuous models-with and without harvesting-are reviewed. Both deterministic and stochastic models are...

https://www.simiode.org/resources/4176

2001-Tsoularis-Analysis of logistic growth models

By A. Tsoularis

Edit

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Category

Potential Scenario Ideas

Published on

20 Jun 2015

Abstract

Article Review and Annotation

Tsoularis, A. 2001. Analysis of logistic growth models, *Research Letters in the Information and Mathematical Sciences*. 2:23-46. Full article available at http://mro.massey.ac.nz/handle/10179/4341 .

Article Abstract: A variety of growth curves have been developed to model both unpredated, intraspecific population dynamics and more general biological growth. Most successful predictive models are shown to be based on extended forms of the classical Verhulst logistic growth equation. We further review and compare several such models and calculate and investigate properties of interest for these. We also identify and detail several previously unreported associated limitations and restrictions.

The paper presents an historical development of the logistic equation in its various forms, including Verhulst, Pearl and Reed, Gompertz, Bertalanffy, Richards, and others. For each type of equation there is a section on analysis to include broad general behaviors and qualitative discussions. The issues usually are around the variations and justifications as well as resulting analyses for different functional response terms, $f(N)$, in the general logistic equation:

$$N'(t) = r\, N(t)\, f(N(t)).$$

Technique Narrative

3-090-T-Text-ChebyshevPolynomialSolution

By Gabriel Costa

Mathematical Sciences, United States Military Academy, West Point NY USA

Edit

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Category

Technique Narratives

Published on

03 Jun 2015

Abstract

The Chebyshev equation, $(1 - x^2)\, y'' + A\, x\, y' + B\, y = 0$, presented as a vehicle to view series solutions techniques for linear, second order homogeneous differential equations with non-constant coefficients. This project requires the student to investigate properties enjoyed by special solutions to this equation known as Chebyshev Polynomials, while also learning something about the person for whom the equation is named.

Technique Narrative

7-006-S-Text-LaplaceTransformBirth

By Sania Qureshi

Basic Sciences and Related Studies, Mehran University of Engineering and Technology, Jamshoro, Sindh PAKISTAN

Edit

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Category

Technique Narratives

Published on

24 Feb 2016

Abstract

We present a way of introducing the Laplace Transform as the continuous analogue of a power series expression of a function.

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SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

TEACHER VERSION

Inegrating Factor

oops

Brian Winkel, Director
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BrianWinkel@simiode.org

Abstract: We develop a strategy to solve first order differential equations by transforming one side of the equation to the derivative of a product of two functions, thereby making it easy to antidifferentiate that side. We then have to face the other side of our modified differential equation and this still could be difficult or impossible to antidifferentiate. Despite this, the method works for a number of differential equations which are reasonable models of phenomena.

Keywords: First order, differential equation, integrating factor, technique,models

Tags: method, solution strategy, tank, RL circuit,

INTRODUCTION

We develop and discuss a strategy to solve first order, ordinary differential equations using a technique based on the product rule for differentiation. It is called *integrating factor*. Basically we build a function (called an *integrating factor*) with which we multiply both sides of our differential equation we wish to solve so that one side looks just like the derivative of a product of two functions. This makes one side of the differential equation easy to anti-differentiate, but we have to address the other side of the differential equation which we just multiplied by the integrating factor. There are modeling situations which lead to such equations and we use one involving electrical circuits to motivate the method.

In Figure 1 we show a RL circuit:

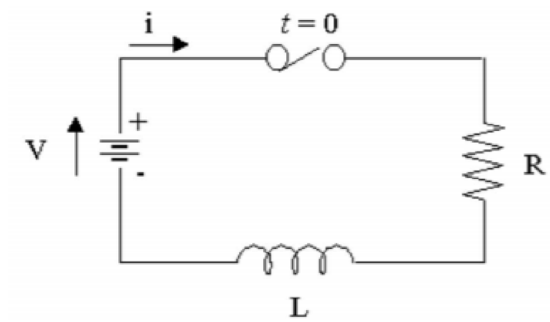

Figure 1. Electric circuit in which the current $i = I(t)$ is flowing across a resistor R measured in Ohms and an inductor L measured in Henrys. The current is a flow of negative electrons. A switch is at the top and the indication is that at time $t = 0$ we through the switch on, thus enabling the current to flow through the circuit.

Here is the differential equation which we will show later models the current $I(t)$ going through the circuit containing a resistor, R, and an inductor, L:

$$L\frac{dI}{dt} + R \cdot I(t) = E(t)\,. \tag{1}$$

Just the visualization of (1) hints (or begs, depending upon how well you remember your differentiation formulae) to be seen as the derivative of a product. Look see in (2).

$$\overbrace{L\frac{dI}{dt} + R \cdot I(t)}^{\text{looks like } u\cdot\frac{dv}{dt}+\frac{du}{dt}\cdot v}, = E(t)\,. \tag{2}$$

Search

resources:modeling_scenarios:Torricelli

Search

RESULTS (PAGE 1 OF 2)

Torricelli's Law   Torricelli

6-030-T-SaltTorricelli

Modeling Scenarios 5 Jun 2015 Contributor(s): Brian Winkel

We build on a model (Torricelli's Law) for the height of a falling column of water with a small hole in the container at the bottom of the column of water.

https://www.simiode.org/resources/815

1-058-T-WaterClocks

Modeling Scenarios 27 Nov 2016 Contributor(s): Sania Qureshi, Brian Winkel

We apply Torricelli's Law to the task of building a water clock in which the height of the water in a container falls at a constant rate when the container has a hole in the bottom to let the water flow out.

https://www.simiode.org/resources/3144

1-058-S-WaterClocks

Modeling Scenarios 27 Nov 2016 Contributor(s): Sania Qureshi, Brian Winkel

We apply Torricelli's Law to the task of building a water clock in which the height of the water in a container falls at a constant rate when the container has a hole in the bottom to let the water flow out.

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SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

TEACHER VERSION
BUILDING WATER CLOCKS

Sania Qureshi
Basic Sciences and Related Studies
Mehran University of Engineering and Technology
Jamshoro, Sindh PAKISTAN
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Brian Winkel
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oops

Abstract: We apply Torricelli's Law to the task of building a water clock in which the height of the water in a container falls at a constant rate when the container has a hole in the bottom to let the water flow out. First, we review the principles and derivation in Appendix from [2] of the applicable physics in Torricelli's Law. Second, we determine the shape of the container, given a constant spigot size for the exit hole, so that the water height falls at a constant rate.

Keywords: water clock, flow, spigot, Torricelli's Law

Tags: first order, differential equation, nonlinear

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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

TEACHER VERSION

MODELING THE SMOKING PROCESS OF SOUTHERN BARBECUE

Troy Henderson
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University of Mobile
Eight Mile AL 36613 USA
thenderson@umobile.edu

Abstract: We offer raw data collected from two thermometers used in the smoking process of Southern barbecue. One thermometer measures the temperature inside of the smoke chamber and the other measures the internal temperature of the meat. This data can be used to model and predict the amount of time required to smoke meats for barbecue, but it can also be used to justify the existence of and quantify the temperatures that constitute a "stall" when smoking large cuts of meat.

Keywords: Barbecue, Newton's law of cooling/warming, modeling, logistic function

Tags: first order, ordinary differential equations, non-linear regression

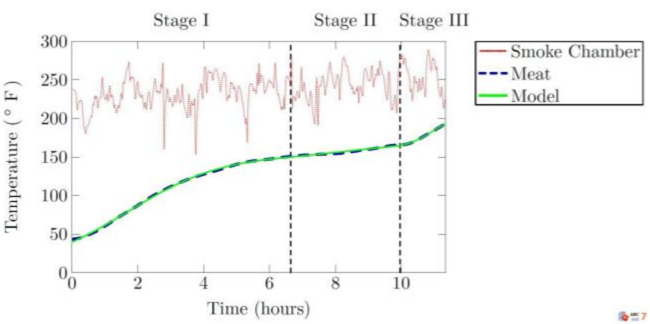


Figure 9. Temperature model fit in each stage

Figure 2. Untrimmed Brisket.

Figure 3. Trimmed/Separated Brisket.

Figure 4. Brisket and Shoulder.

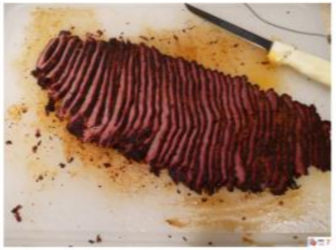
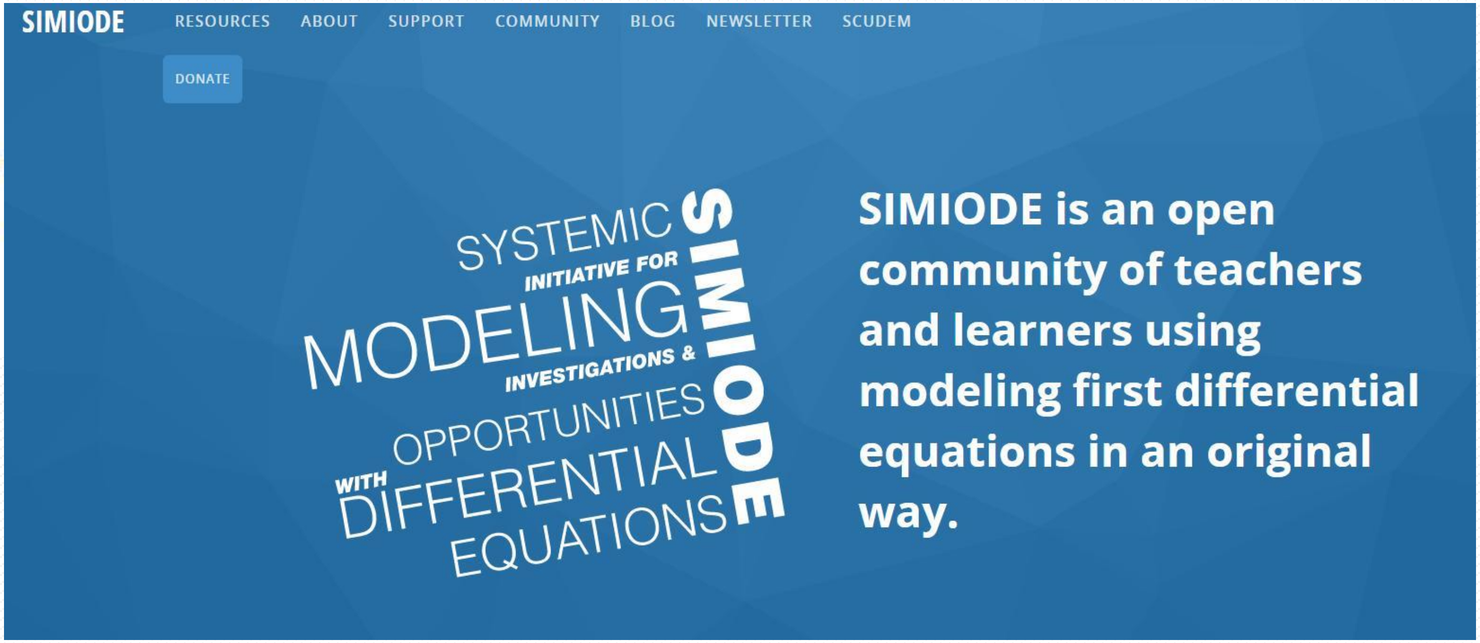


Figure 5. Sliced Brisket.

Figure 6. Finished Brisket.

A bit of history

- SIMIODE founded in Spring 2013 with private funding
- SIMIODE became 501(c)3 nonprofit educational organization in Summer 2016 - support from corporations and individuals
- SIMIODE received National Science Foundation Division of Undergraduate Education Grant in Spring 2018 for three year support



Consider the question

"What can you learn from doing
modeling in a differential equations course?"

Short, but essentially complete answer, "Lots!"

The **key** is that you are always reaching out for applications, seeking models and data, and bringing them back to your class as lessons or projects.

In fact, I wrote a paper about this key activity in a distinguished journal . . . . . .

PRIMUS, 23(3): 274–290, 2013

ISSN: 1051-1970 print / 1935-4053 online
DOI: 10.1080/10511970.2012.753966

Taylor & Francis
Taylor & Francis Group

Browsing Your Way to Better Teaching

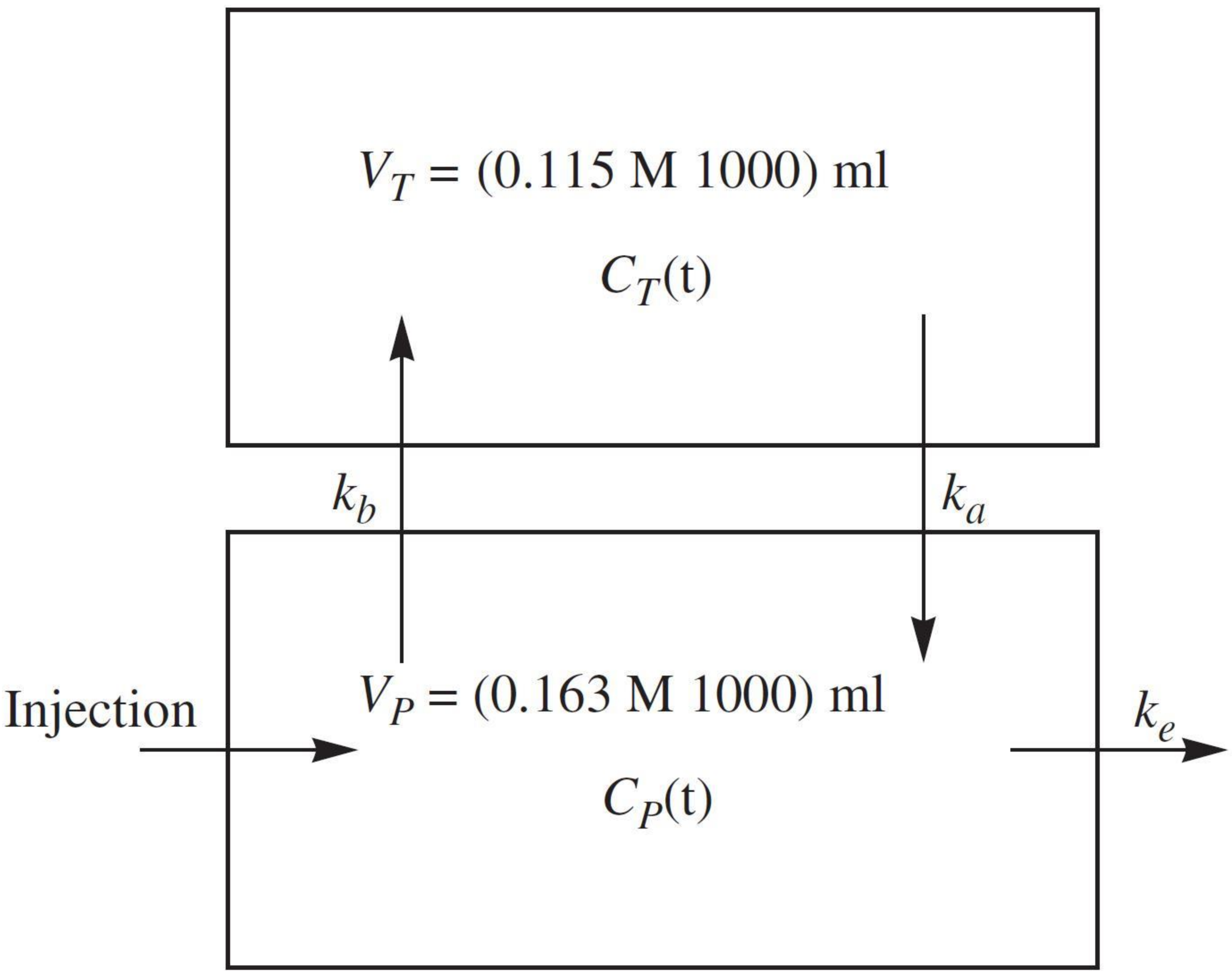
Brian Winkel

Abstract: We describe the use of browsing and searching (in libraries, online, inside sources, at meetings, in abstracts, etc.) as a way to stimulate the teacher of undergraduate mathematics, specifically in differential equations. The approach works in all other areas of mathematics. Browsing can help build new and refreshing teaching materials based on how mathematics is used and explored in places other than mathematics. These "other" places are where almost all of our students will be going after they study with us and we should: (i) know about their journey and arrival points; and (ii) understand the disciplinary approaches for those areas which sent these students to us in the first place for their mathematics studies. We describe a personal browsing experience that spanned almost 40 years and proved to be very worthwhile in finding applications of differential equations to modeling Lysergic Acid Diethylamide in the human body.

Keywords: Browsing and searching, sources, mathematical modeling, differential equations, compartment model, pharmacokinetics, Lysergic Acid Diethylamide (LSD).

	Time (hr)	0.833	0.25	0.5	1.0	2.0	4.0	8.0
Subject 1	Plasma Conc (ng/ml)	11.1	7.4	6.3	6.9	5.	3.1	0.8
	Perform Score (%)	73	60	35	50	48	73	97
Subject 2	Plasma Conc (ng/ml)	10.6	7.6	7.	4.8	2.8	2.5	2.
	Perform Score (%)	72	55	74	81	79	89	106
Subject 3	Plasma Conc (ng/ml)	8.7	6.7	5.9	4.3	4.4	—-	0.3
	Perform Score (%)	60	23	6	0	27	69	81
Subject 4	Plasma Conc (ng/ml)	10.9	8.2	7.9	6.6	5.3	3.8	1.2
	Perform Score (%)	60	20	3	5	3	20	62
Subject 5	Plasma Conc (ng/ml)	6.4	6.3	5.1	4.3	3.4	1.9	0.7
	Perform Score (%)	78	65	27	30	35	43	51

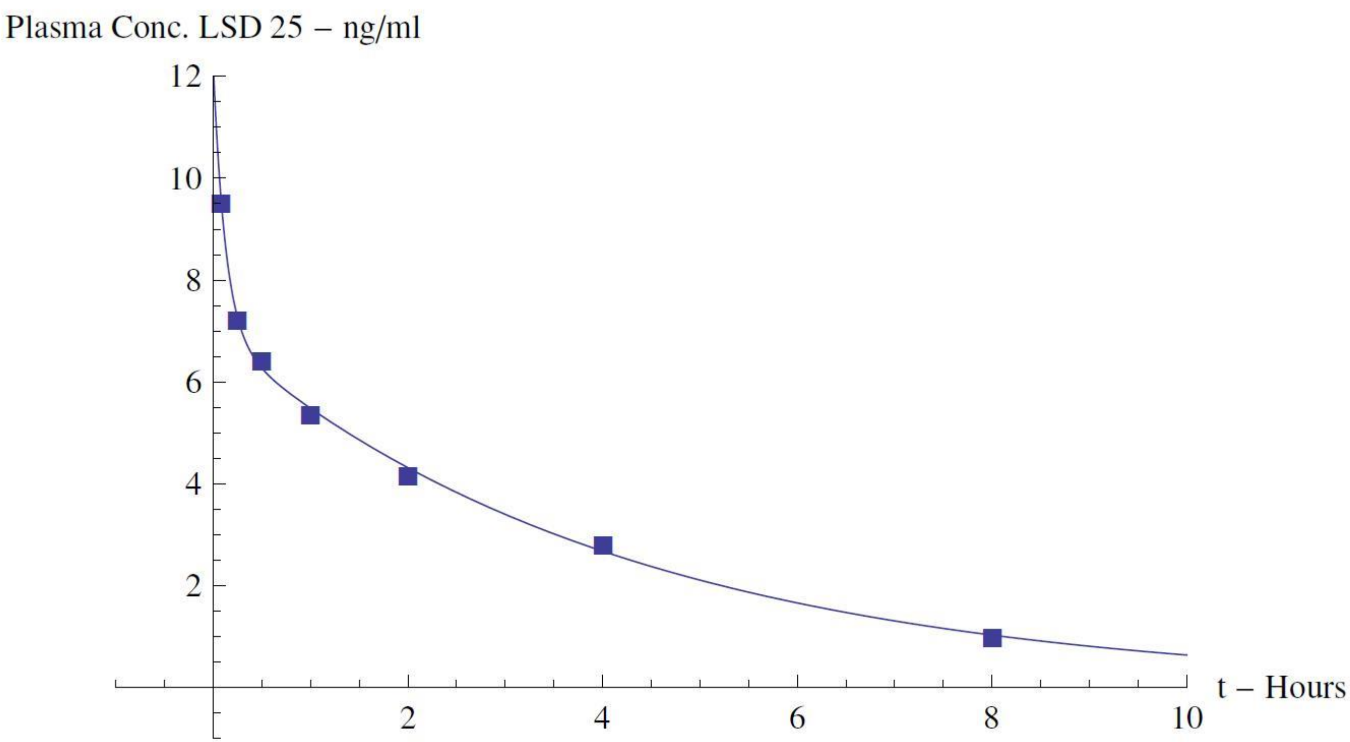
Table 1. Summary of data collected [1, 14] on 5 male volunteers who were administered LSD and then tested on performance (Perform Score (%)) on simple addition questions. Both performance Score and Plasma Concentrations of LSD were recorded at 5, 15, 30, 60, 120, 240, and 480 minutes after the initial infusion of LSD.



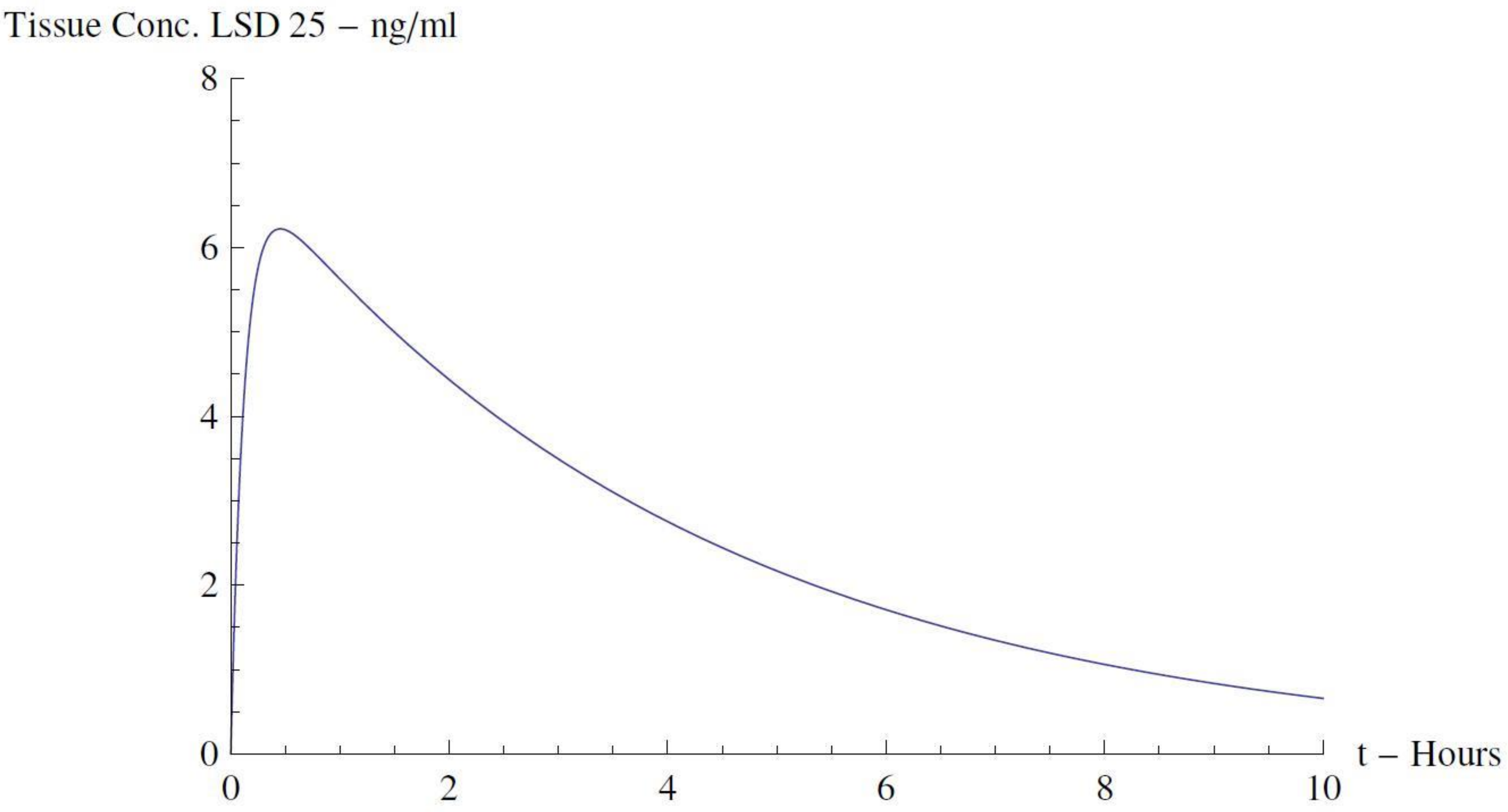
$V_T = (0.115\ M\ 1000)$ ml
$C_T(t)$
k_b
k_a
Injection
$V_P = (0.163\ M\ 1000)$ ml
k_e
$C_P(t)$

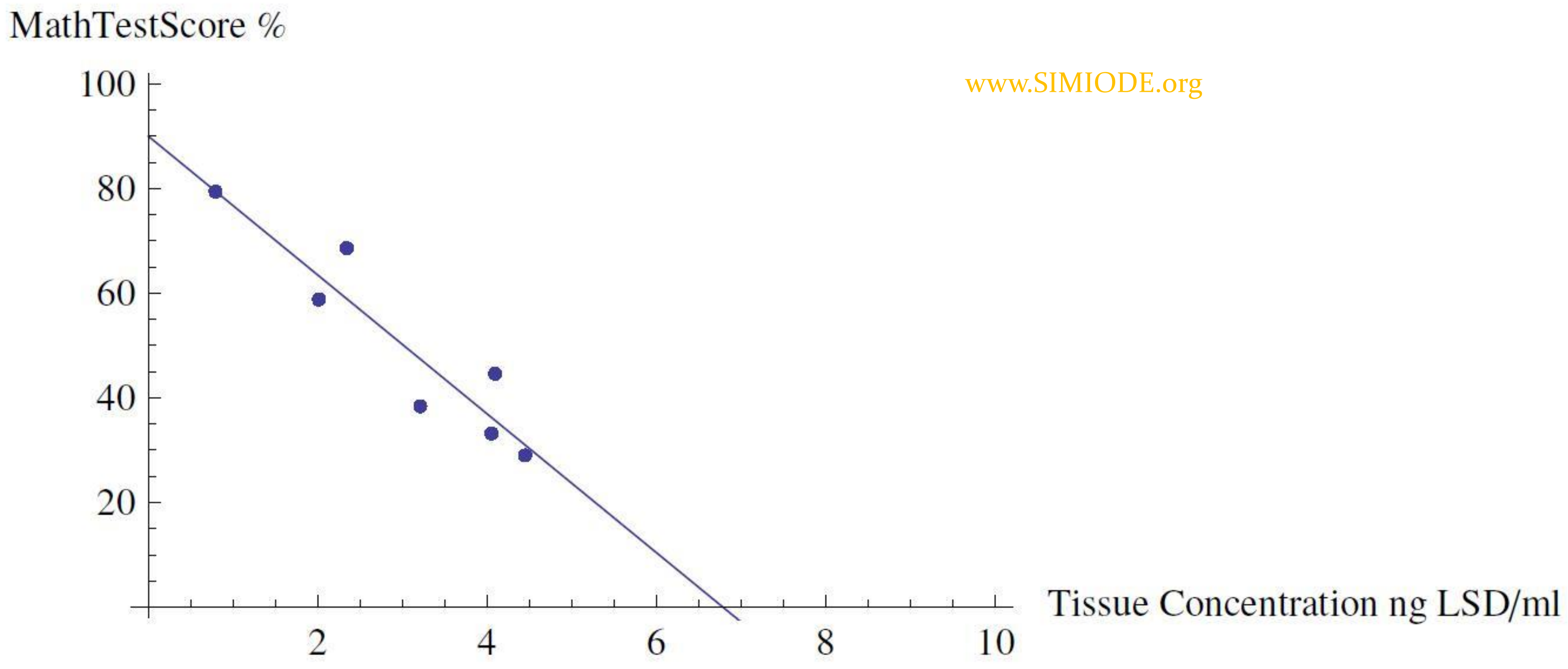
$$
\begin{aligned}
V_P C'_P(t) &= k_a V_T C_T(t) - k_b V_P C_P(t) - k_e V_P C_P(t)\\
V_T C'_T(t) &= k_b V_P C_P(t) - k_a V_T C_T(t)\,.
\end{aligned}
$$

$$SSE(k_a, k_b, k_e) = \sum_{i=1}^{7} (C_P(t_i) - O_i)^2$$



$$C_T(t) = 0.128905\left(55.419e^{-0.238492t} - 55.419e^{-7.99617t}\right) .$$




MathTestScore %
100
80
60
40
20
2
4
6
8
10
Tissue Concentration ng LSD/ml
www.SIMIODE.org

Reaching out to local industry colleagues . . . .

. . . led to three hour workshop by chemist and mathematician from The Upjohn Company on pharmacokinetics.

Brought STEM faculty AND students together for active learning, working session on modeling and parameter estimation

Metzler, C. M. 1969. A mathematical model for the pharmacokinetics of LSD effect. *Clinical Pharmacology and Therapeutics*. 10(5): 737–740.

Metzler, C. M. and G. L. Elfring. 1978. Letter to the Editor: Curve fitting and modeling in pharmacokinetics: a response. *Journal of Pharmacokinetics and Pharmacodynamcs*. 6(5): 443–446.

Other interactions can occur

While at the United States Military Academy at West Point NY

Director of MA364 – Engineering Mathematics

Team of mathematics AND engineering faculty in teaching team.

LTC Keith Landry, fresh from his PhD in Civil Engineering at RPI, joined the team.

Keith introduced Tuned Mass Dampers to the course

Examples of Tuned Mass Dampers

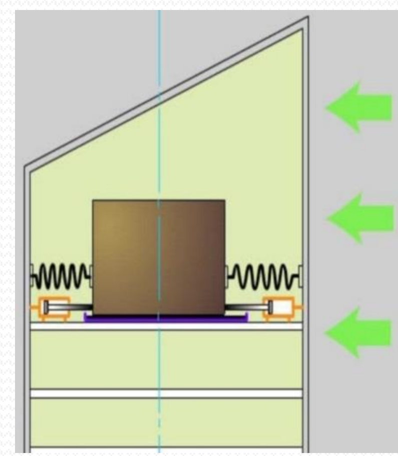
TMD: Schwedter Strasse, Berlin

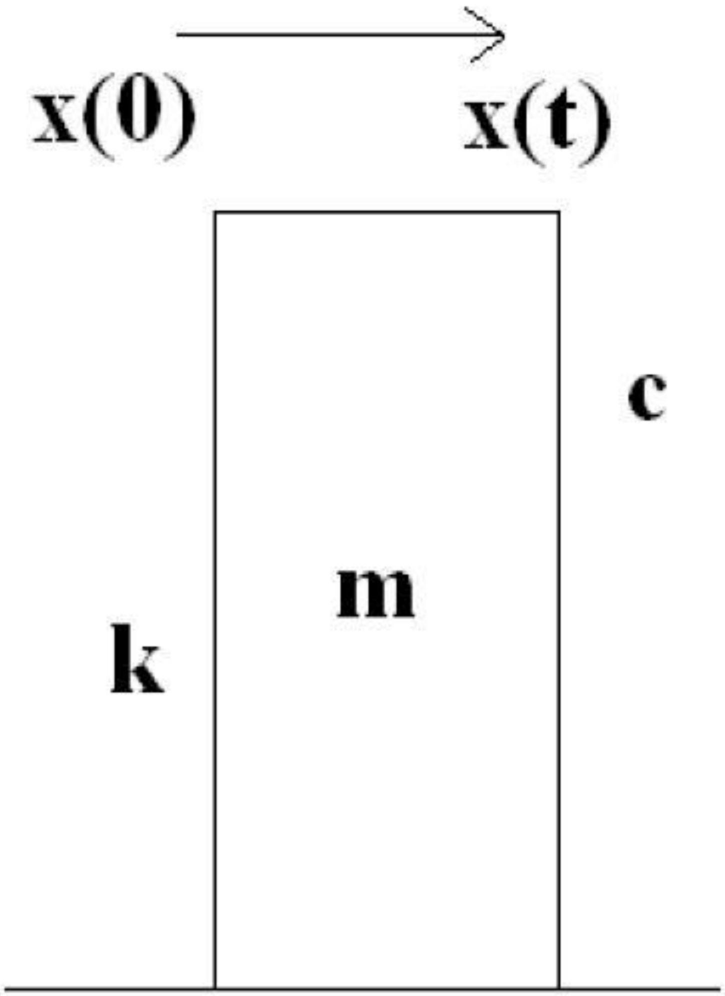
TMD: Skywalk @ Grand Canyon

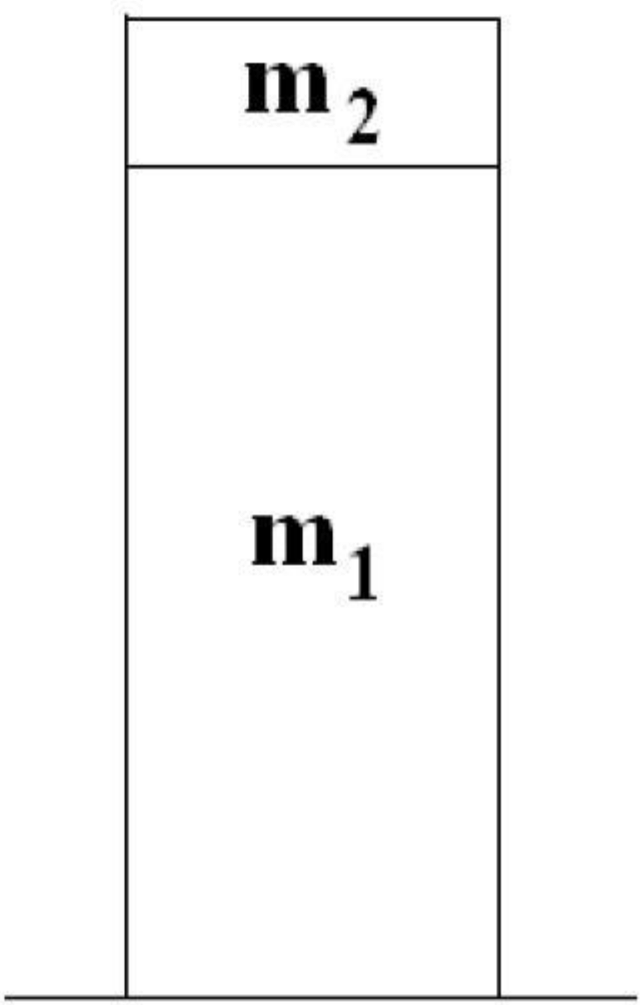
TMD: Stockbridge Damper

TMD: Taipei 101

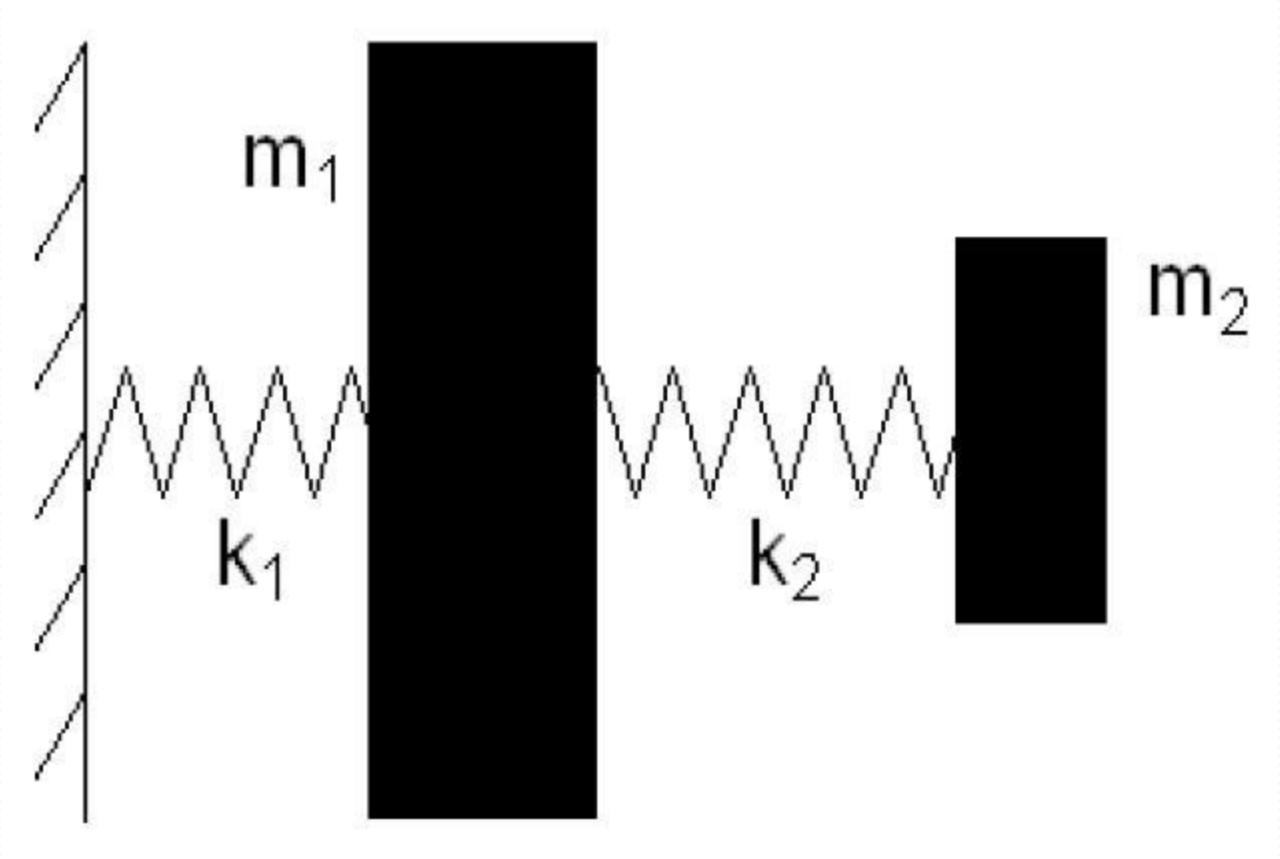
TMD: Citicorp Building







Structure sways? Introduce second mass, but tune it.



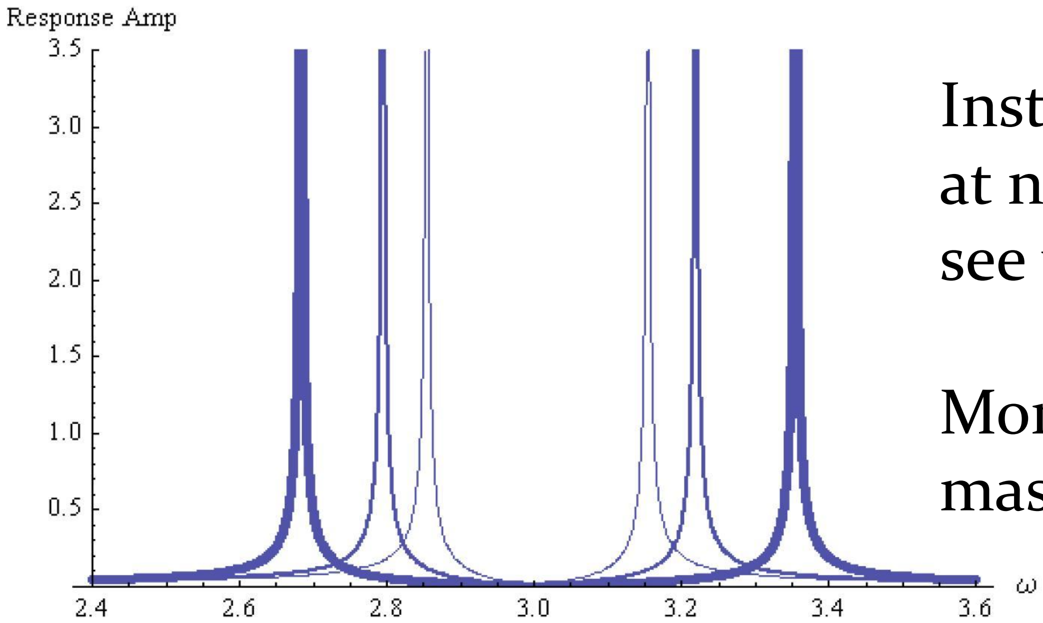
Ideal Tuned Mass Damper in which there is no resistance.

Make natural frequency of added mass m_2
the same as that of original mass m_1 .

We build a system of differential equations using FBD for displacement of each mass.

$$m_1 x_1''(t) + k_1 x_1(t) + k_2 x_1(t) - k_2 x_2(t) = F_0 \cos(\omega t),$$

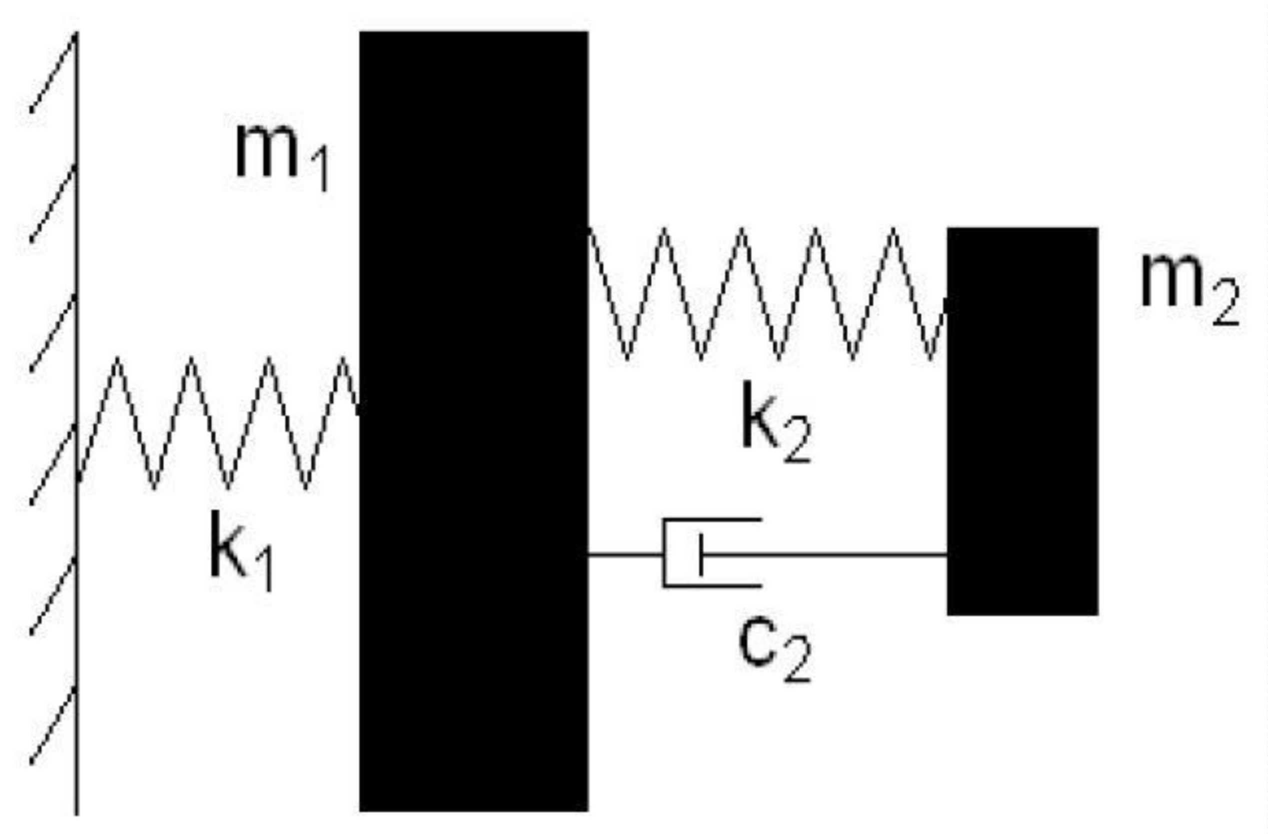
$$m_2 x_2''(t) - k_2 x_1(t) + k_2 x_2(t) = 0\,.$$



Instead of possible resonance at natural frequency ω = 3 we see total damping.

Moreover, the bigger second mass is the wider the coverage.

We could analyze more realistic models with resistance to one or both masses.



Time (s)	0	0.347	0.47	0.519	0.582	0.65	0.674	0.717	0.766	0.823	0.87	1.031	1.193	1.354	1.501	1.726	1.873
Distance (m)	0	0.61	1.00	1.22	1.52	1.83	2.00	2.13	2.44	2.74	3.00	4.00	5.00	6.00	7.00	8.50	9.50

Table 1. Time and distance traveled data on a free falling shuttlecock which is dropped from a height of 2 m at rest.

Source: Peastrel, M., R.Lynch, and A. Armenti, Jr. 1980. Terminal velocity of a shuttlecock in vertical fall. *American Journal of Physics*. 48(7): 511-513.

Many sources on falling body with resistance. Use some with real data or generate the real data with students.

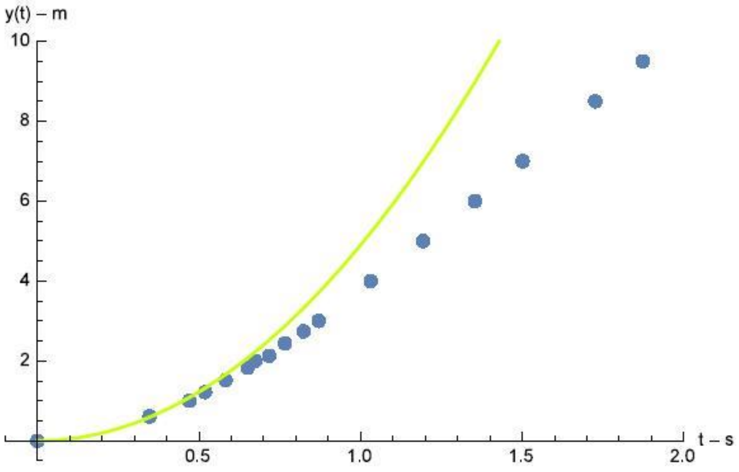


Figure 3. Plot of the model with no resistance, $m * v'(t) = m * g$, and the original data.

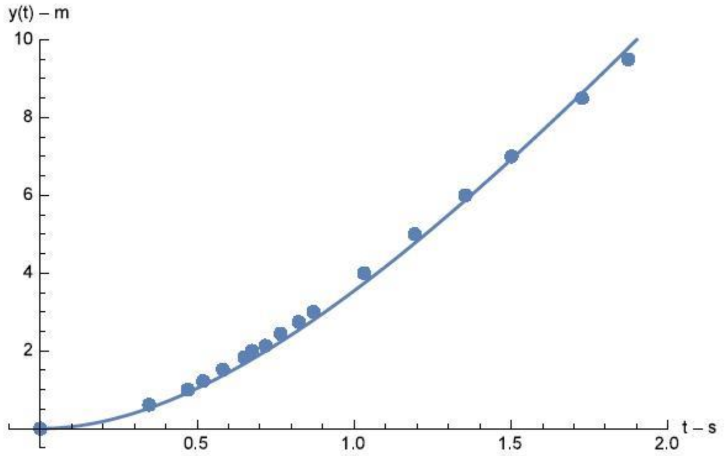


Figure 4. Plot of the model with resistance using $r = 1$, $v'(t) = -k * v(t)^1 + g$, and the original data.

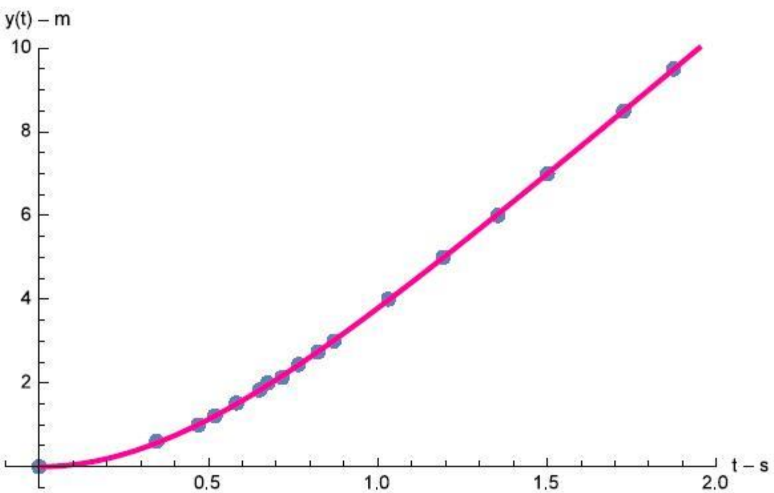


Figure 6. Plot of the model with resistance using $r = 2$, $v'(t) = -k * v(t)^2 + g$, and the original data.

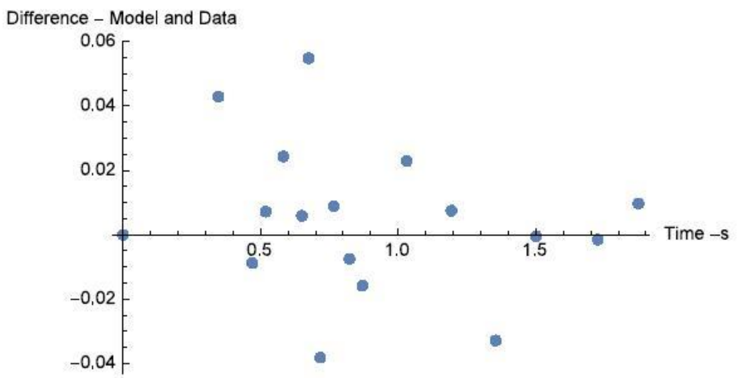


Figure 7. Plot of the residuals(differences between model and data) from the model with resistance using $r = 2$, $v'(t) = -k * v(t)^2 + g$, and the original data.

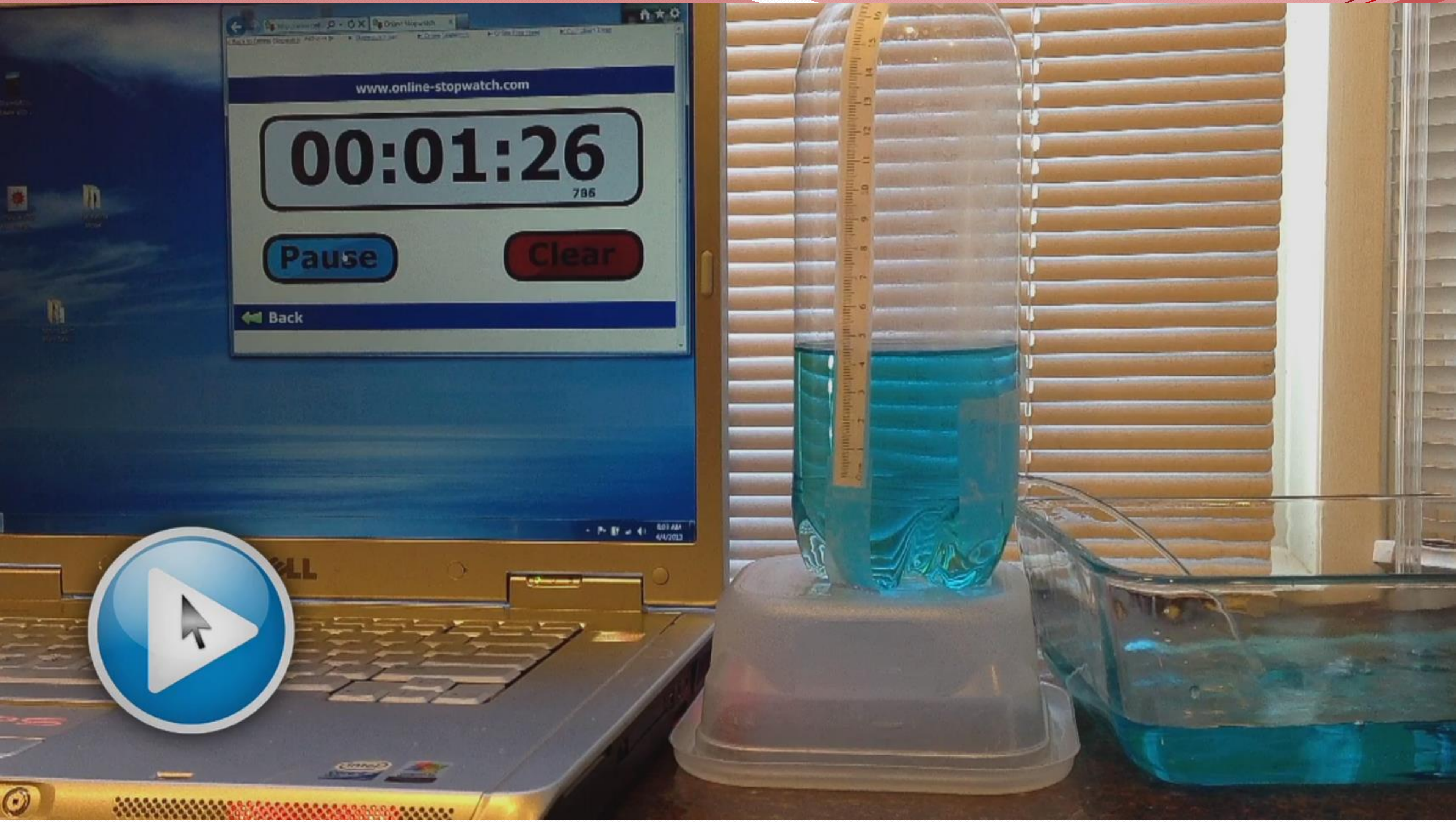
Model	r	$k = a$	SSE	AIC	
Source	2		0.00920356	**-123.811**	
No Resistance		0	128.369	128.369	
Case	$r = 1$	1	1.06174	0.523775	-55.1584
Case	$r = 2$	2	0.211834	0.00920356	**-123.863**
General r	2.019	.0205	0.00919924	**-121.871**	

Table 2. Information we need about our models to decide which model is best is presented.

$$AIC = 2(1 + k) + n \log \left(\frac{\text{RSS}}{n} \right) .$$

k is # parameters and
n is # data points

Akaike information criterion (AIC)



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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

TEACHER VERSION
CIRCUIT TUNER

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Abstract: We present essential definitions and laws for the study of simple RLC electrical circuits and build a differential equation model using these notions. We describe how such a circuit can be used to tune a radio to a certain input frequency.

Keywords: circuit, tuner, nonhomogeneous, second order, Kirchhof's Voltage Law

Tags: gain, resistance, inductance, capacitor, amplitude,differential equation, model, driver, voltage, current, frequency response, maximum

Kirchhoff's Voltage Law says that the sum of the voltages in a circuit is equal to that of the induced voltage, *Emf(t)*. Here *Q(t)* is charge and *I(t)* = *Q'(t)* is current.

(C) The voltage (in volts) drop, E_C, across a capacitor rated at C *farads* is

$$E_C = \frac{1}{C} \cdot Q(t) .$$

(R) The voltage (in volts) drop, E_R,across a resistor rated at R *ohms* is

$$E_R = R \cdot \frac{dQ(t)}{dt} = R \cdot I(t) .$$

(L) The voltage (in henry) drop, E_L, across an inductor rated at L *henrys* is

$$E_L = L \cdot \frac{dI(t)}{dt} .$$

$$E_C + E_R + E_L = \frac{1}{C} \cdot Q(t) + R \cdot \frac{dQ(t)}{dt} + L \cdot \frac{dI(t)}{dt} = \mathrm{Emf}(t)$$

Differentiate both sides. . .

$$\frac{1}{C} \cdot \frac{dQ(t)}{dt} + R \cdot \frac{dQ^2(t)}{dt^2} + L \cdot \frac{dI^2(t)}{dt^2} = \mathrm{Emf}'(t)$$

. . . and rearrange.

$$L \cdot \frac{dI^2(t)}{dt^2} + R \cdot \frac{dI(t)}{dt} + \frac{1}{C} \cdot I(t) = \mathrm{Emf}'(t)$$

$$\mathrm{Emf}(t) = \sin(wt)$$

Input voltage Emf(t) yields

circuit differential equation with *y(t)* = *I(t)*.

$$L \cdot y''(t) + R \cdot y'(t) + \frac{1}{C} \cdot y(t) = \mathrm{Emf}'(t) = \omega \cos(\omega t)$$

As with spring mass systems we can study amplitude response of current to frequency of input voltage.

Further one can then compute Gain, i.e., ratio of the amplitude of the steady state output voltage to the amplitude of the input voltage.

We note amplitude of input voltage Emf(t) = sin(ω t) is 1. Thus gain IS the amplitude of the steady state solution.

$$\text{Amplitude}(y_{\text{steady state}}(t)) = R\sqrt{\frac{C^2\omega^2}{C^2\omega^2\left(L^2\omega^2+R^2\right)-2CL\omega^2+1}}$$

Suppose L and R are fixed and we can vary capacitance C.

Then for given input frequency (presumably from the “ether” as a radio broadcast signal) we can find C which maximizes gain.

That is exactly what we do in radio tuner.
We tune by finding C which maximizes gain.

$$\text{gain} = R\sqrt{\frac{C^2\omega^2}{C^2\omega^2\left(L^2\omega^2 + R^2\right) - 2CL\omega^2 + 1}}$$

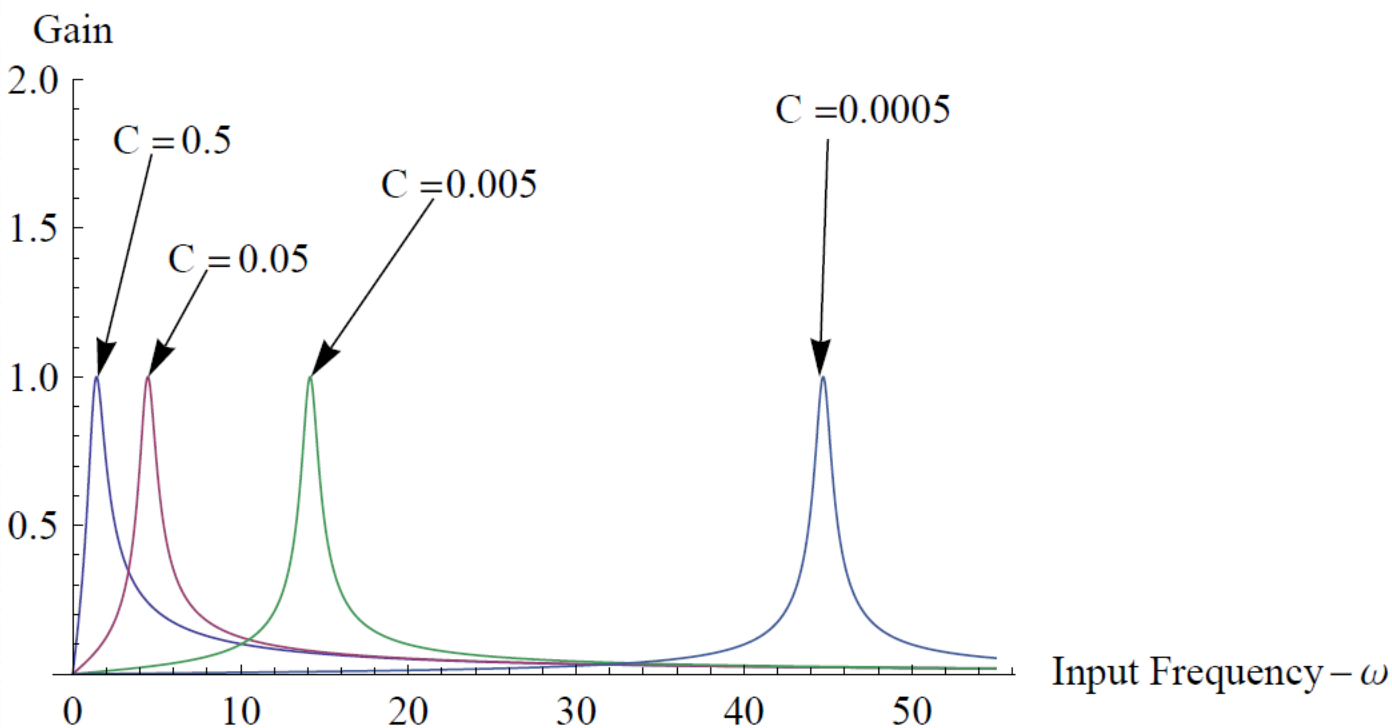
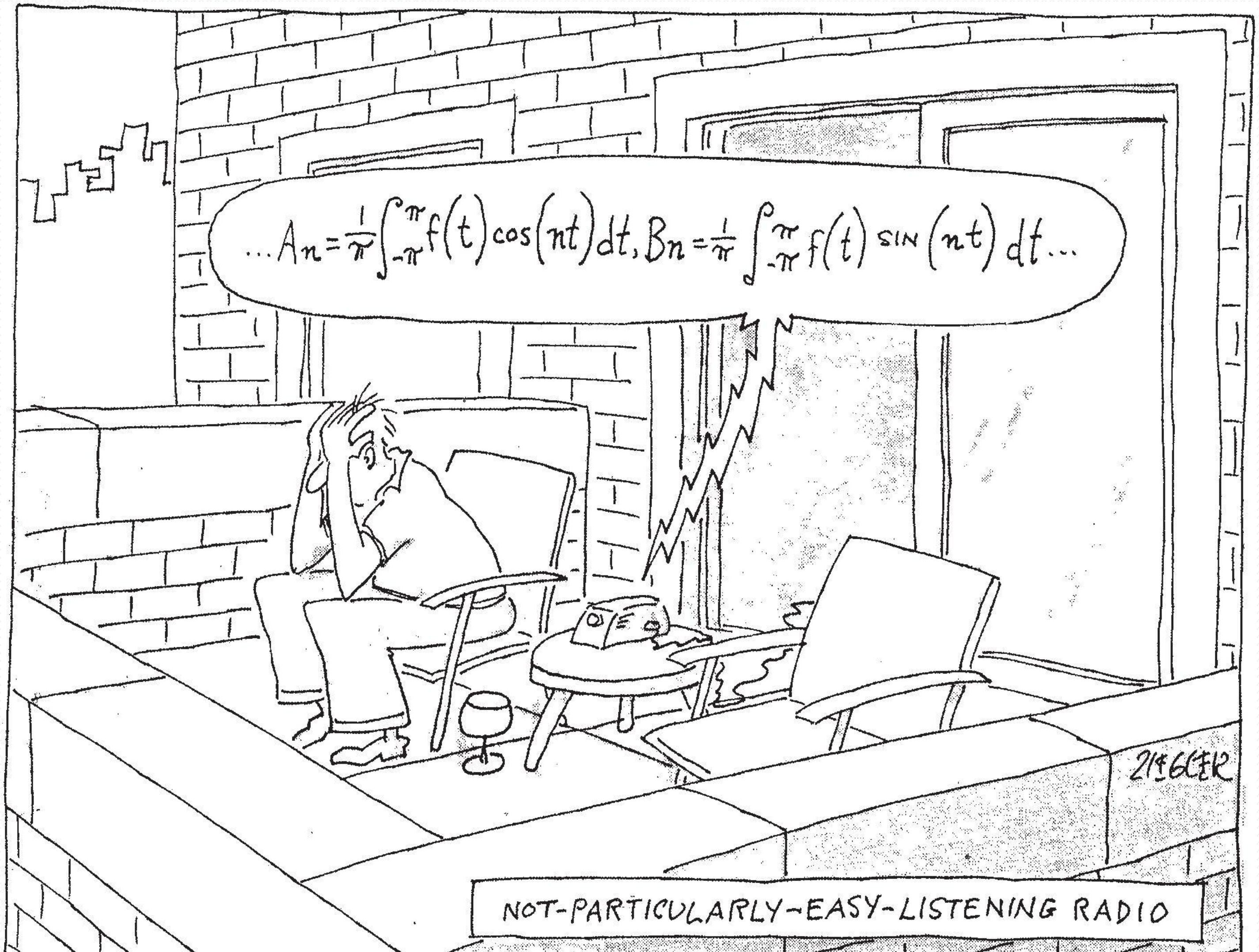


Figure 5. Plot of gain as a function of input frequency, ω, for various values of capacitance, $C = 0.5, 0.05, 0.005, 0.0005$ farads in (10) with R at 1 ohm and L at 1 henry.

Not all work. . . not all fun. . . building for modeling-first differential equations



These images are related!

$$\sum_{n=0}^{\infty} A_n$$

Finally, in answer to the question

> “What can you learn from doing modeling in a differential equations course?”

we say, “lots” and “you never know.”

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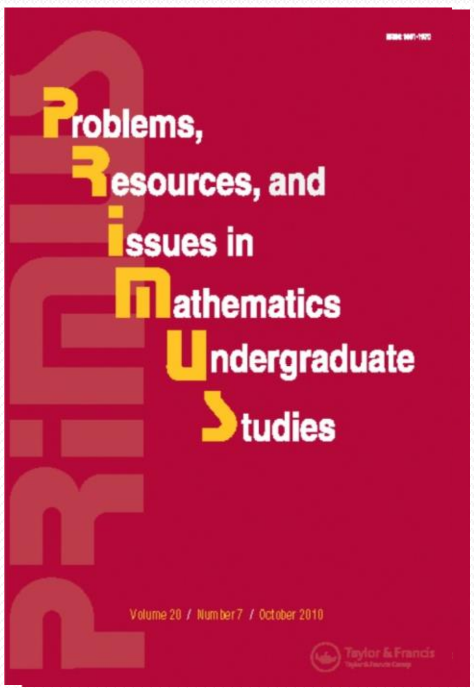

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