

**Using Online Resources**  
**Joint Mathematics Meetings – Baltimore**  
**9:40 AM, Friday, 17 January 2014,**  
**Room 50, BCC**

**Creating and using Online Community**  
**Resources for Teaching Differential Equations**  
**with Modeling and Technology**

**SiMiODE**

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## Abstract

We are building an on-line presence [www.simiode.org](http://www.simiode.org) in support of an effort to offer an approach for teaching differential equations through modeling and technology.

This is **SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations.**

The major thrust of SIMIODE is to fully engage students in modeling scenarios which lead to differential equations study and use. This is coupled with ever present technology use and on-line material fully supporting the learning community. This will no longer require expensive texts.

We believe such modeling scenarios can enrich any class in differential equations.

# SIMIODE

SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS AND OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

## Welcome To SIMIODE

**SIMIODE** - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations is a fully supportive community of teachers and learners. **SIMIODE** provides an open environment for teaching and learning differential equations using modeling and technology throughout the entire experience. Rich modeling scenarios, data sets, and videos will motivate and support the teaching and learning and there will be ample traditional text readings and exercises as well. All teaching material in **SIMIODE** is built from best learning practices by members of the **SIMIODE** community. The HUB nature of **SIMIODE** provides for collaborative teaching and learning within and across campus coursework.

Files presented here are articles about SIMIODE approach and NOT actual SIMIODE lesson scenarios or text material. These are under construction and review.

## SIMIODE Videos

- [Video For Data Collection In Support Of Modeling With Torricelli's Law](#)



- [Click Here To See Video Of Modeling Approach To Analysis Of Spring Mass Damper.](#)

## Menu

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**SIMIODE** Active  
HUB Available In  
Winter 2013

And now for something totally different . .

As part of potential pathway through resources consider the following introduction.

## **Modeling Chapter One**

### **First Order Differential Equations**

#### **Narrative and Development**

Modeling Death with M&M 's and Simulation

Immigration Model with M&M's

Modeling Change Discretely

Moving to Differential Equation Model

## Equipment:

- 1 small bag regular M&M's
- 1 small cup
- 1 level surface

## Early actions:

- Open bag
- Count out 50 M&M's
- Place M&M's in cup
- Gently toss M&M's onto surface
- Count and remove all M&M's  
with m facing up – they die!



Add 10 immigrant M&M's.

Later action:  
Yummy!!!

# Generation 6 toss



Early on in the teaching, through physical simulation of a “real” population, students collect data for a mathematical model of the same phenomenon, build mathematical models (difference and differential equations), and then compare the results model to data and of the two models.

Exponential decay for simple death model is natural concept.

Equilibrium value due to immigration model is natural concept.

Students develop model and criterion for assessing just how good the model is with respect to predicting the behavior observed.

The opportunity is rich with many parallels to modeling in the real world and the students can see at each step the assumptions, data collection, modeling, comparisons, and validation.

## **Activities or Modeling Scenarios**

Modeling Immigration in a Petri Dish

Bank Investment Analysis and Loan Analysis

Saving for Child's College Education

Evaporation and Radioactive Decay Modeling

Modeling Sphere of Salt in Solution

Modeling the Spread of Oil Slick with  
Incomplete Data

Hang Time Modeling

Modeling the Emptying of a Column of Water

Chemical Kinetics Models - Zeroth, First, and  
Second Order Reactions

Tunnel Construction by Ants and Humans

Sublimation of Carbon Dioxide

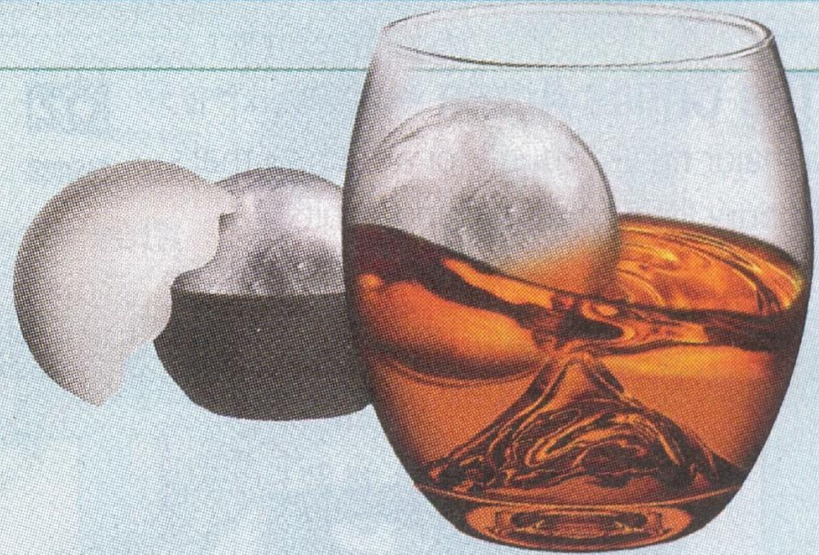


# Best Gifts

GUARANTEED

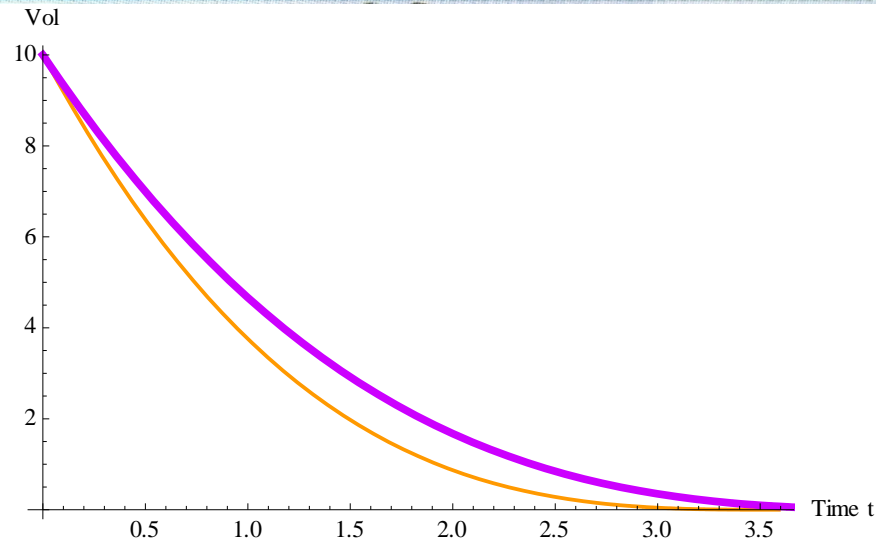
## K. **NEW** On the ROCK Glass with Ice Ball Maker

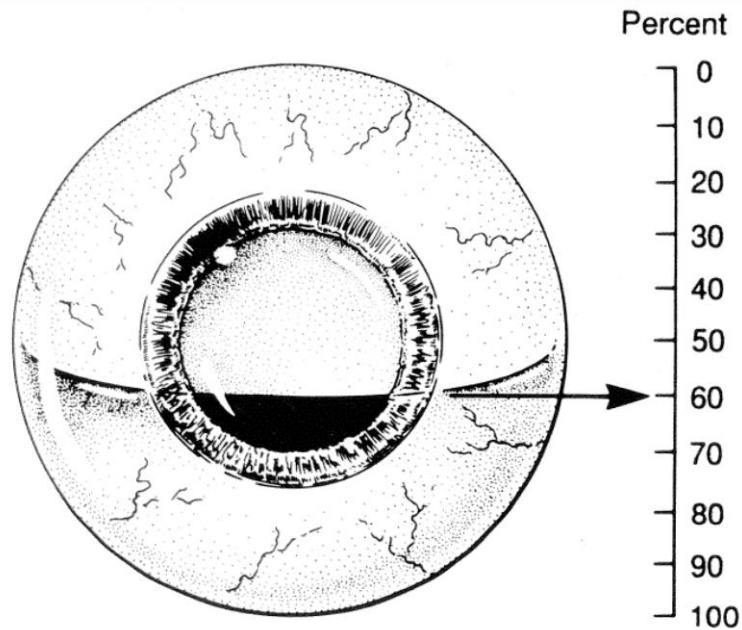
Unique glass makes it easy to swirl ice for better aroma. Lets you make ice balls that outlast cubes and won't water drinks down.



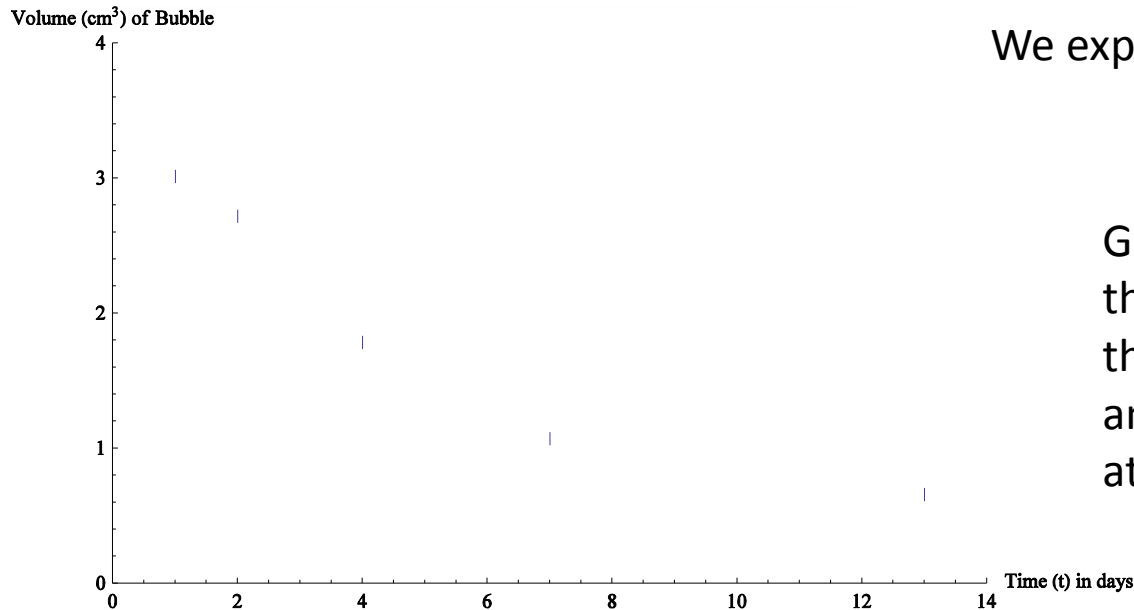
843819 (Includes 1 Ice Ball Mold)	<b>\$19.99</b>
Ice Ball Mold 2-Pack	843822 <b>\$10.99</b>
Gift Set (2 Glasses & 4 Molds)	854465 <b>\$39.99</b>

Build a differential equation for the melting of a sphere and a cube of same volume and based on the assumption that the rate of melting is proportional to surface area we find the following. Purple is sphere and orange is cube.





60% Bubble



In retinal surgery a bubble of gas is placed in the intraocular region of the eyeball to help tampinade the wound.

The ophthalmological profession uses an exponential decay model to predict when the bubble has totally dissipated.

Where did this come from?

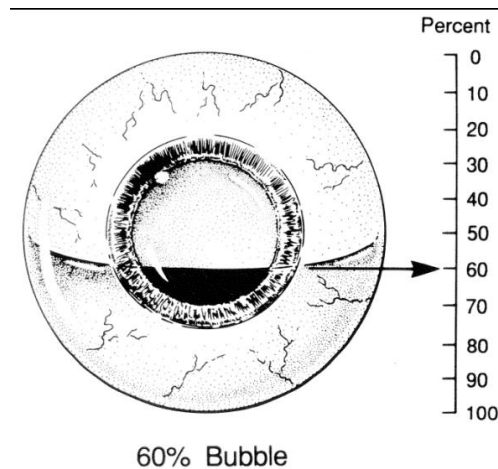
Is that the best and only model?

We explore and compare other models.

Gas may leave the bubble both through the contact surface area with the retina at the top of the bubble and through the fluid in the eyeball at the base of the bubble.

$$\frac{dV}{dt} = -k_1 S(t)^{3/2} - k_2 M(t)^{3/2}, \quad V(0) = V_0$$

Here  $S(t)$  is the surface area of contact of the bubble with the retinal surface and  $M(t)$  is the surface area of the meniscus at the bottom of the bubble.

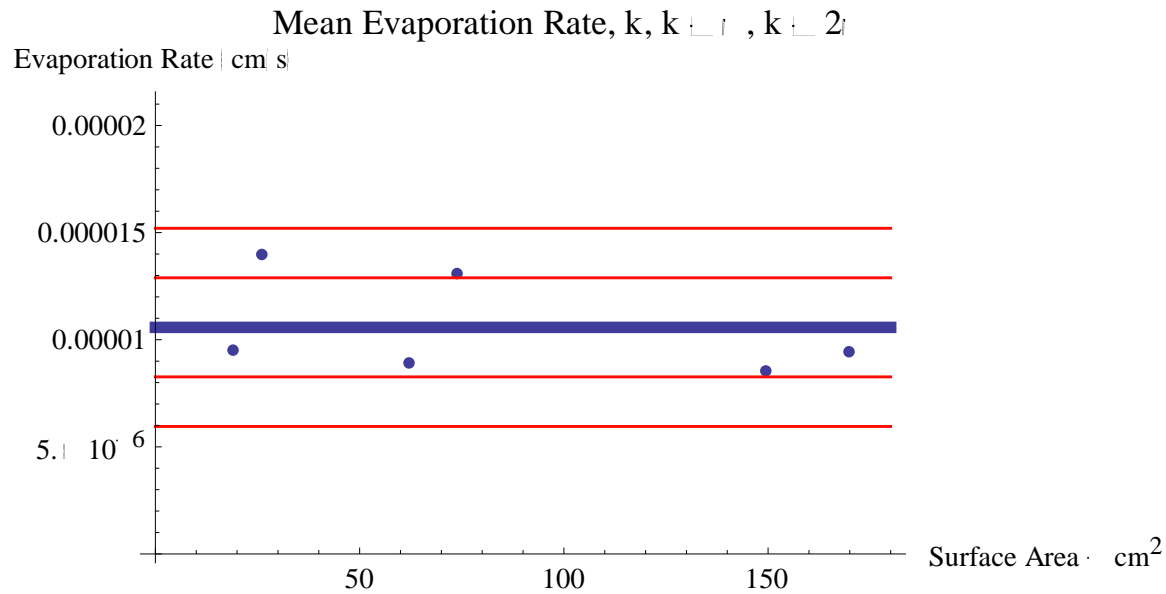


What parameters  $k_1$  and  $k_2$  make the best model and what is a criterion for best?

Evaporation of 90% isopropyl alcohol in various radii of Petri dishes, different surface areas.

$$V'(t) = -k S(t) = -k \pi r^2$$

Summary below.



OR in funnel with varying cross section.

**Goals of SIMIODE**

**Development of SIMIODE**

**Methods of SIMIODE**

**Offerings of SIMIODE**

**Advantages of SIMIODE**

**Opportunities of SIMIODE**

**Uses of SIMIODE**

# Goals of SIMIODE

SIMIODE offers a **rich environment** for teaching and learning differential equations.

SIMIODE is a comprehensive and cohesive approach to the study of differential equations for students that places **modeling and technology use up front and throughout the process** of motivating, doing, teaching, and learning differential equations.

SIMIODE is **NOT a textbook**, but will fully support a course in differential equations or modeling. A student lifetime fee, expected to be under \$50, will assure financial viability of SIMIODE. SIMIODE projects can serve students in other courses.

SIMIODE will permit teachers to **customize and shape an entire course around modeling and technology** use with broad communication and communal interaction tools in a web environment, thus going far beyond current text materials and current efforts to provide modules in modeling with differential equations.

SIMIODE can catalyze a coalescence of teachers and learners who are using SIMIODE, or pieces as engineering mathematics faculty do, into a **web-based (cyber) community** that supports all participants.

# Development of SIMIODE

Built on HUBZero web technology developed by Purdue University under National Science Foundation support for Nano Technology Community.

“HUBzero® is a powerful, open source software platform for creating dynamic web sites that support scientific research and educational activities “

Joomla based software supported by Purdue University HUBZero team.

Annual HUBbub meeting of users and developers.

See <http://hubzero.org/> for more details.

# Methods of SIMIODE

SIMIODE will provide a refereed set of (1) modeling scenarios and (2) text based technique resources to support learning and teaching.

## **Modeling scenarios**

Classroom tested activities which use a modeling situation to motivate the study of differential equations. Often with data or suggested experiments to validate the differential equation model.

For example, build a model of a dialysis machine as an example of a compartment model in the study of systems of differential equations or design a Tuned Mass Damper to mitigate wild oscillation in a device, be it an electric razor, a sloshing washing machine, or a tall skyscraper.

## **Text based technique resources**

Colleague developed ideas and presentations that work to develop skill competencies in solving differential equations and better understanding the role of differential equations in modeling phenomena.

For example, scaffolded exercises and activities with model references to support learning of constant coefficient linear second order homogeneous and non-homogeneous differential equations.



## Methods of SIMIODE . . . More

HUB Zero based SIMIODE will feature group formation – a class, a study group, a project group, a teacher group, etc. through which sharing of ideas, results, data, video, experiments, documents, articles, etc. will enrich learning.

Sample syllabi from faculty who use SIMIODE will be available to show others how SIMIODE may help them in their course.

Student versions with statement and resources only will be in Student section and annotated COMMENTS will be available in the Teacher section.

Colleagues from business, industry, and government will be involved in developing modeling scenarios and taking students to the step beyond. For example, providing patient data for gas bubble dissipation from surgery or presenting different Tuned Mass Dampers – passive and active.

Interest groups on techniques, applications, models, etc. will be available.

Fora for sharing pedagogy, strategies, uses of technology, etc. will provide information to energize faculty to incorporate SIMIODE approach.

## Offerings of SIMIODE

Place where you can share your ideas, thoughts, hopes, and actions in teaching differential equations through modeling and technology.

Place where your students can learn about the rich uses of differential equations in many fields and appreciate the power and application of rate of change modeling.

Resources for teaching and learning.

Community for students (in live groups) and teachers (in sharing pedagogy and material).

# Advantages of SIMIODE

SIMIODE will provide a supportive community for teachers and students.

SIMIODE will contain all the student will need for success in a differential equations course or a modeling course based on differential equation modeling.

This includes traditional textual material on techniques, but also modeling opportunities through which to build understanding as to the why, not just how, of differential equations.

These modeling scenarios are not just double starred exercises at chapter end, nor are they dropped in show and tell activities.

**Modeling scenarios are the place for starting the learning process in all cases.**

SIMIODE will be **FREE** for several years and when fully operational will **cost students \$45 for life time membership** – lifetime because many scenarios and techniques will be of value to students beyond the current course and students beyond the course will see and want to contribute application lessons and modeling scenarios from their experiences.

# Opportunities of SIMIODE

## Students

Save money (lots!) as one time lifetime membership would be \$45, not a \$300 text book.

SIMIODE would be open community with possibilities for students to contribute lesson materials, activities, data sets, project ideas, etc.

See differential equations in context and understand the role of rate of change modeling through history and application.

Use of online data sets and experiments, e.g., running a video which shows a phenomenon, with units and clock, from which data can be collected and then modeled.

Collaboration on projects with students on other campuses with different perspectives and cultures.

Much more . . . .

# Opportunities of SIMIODE

## Faculty

Reduction in guilt at designing courses around high priced texts while moving into a more open structured, peer generated learning environment.

Refereed teaching materials both for use and for professional scholarship.

Access to experiments and data sets not possible due to local constraints.

Real time meeting place for conversations with others teaching differential equations in a SIMIODE approach.

Team teaching across distance and time.

Support from colleagues for development of individual classroom activities.

Sharing “heads up” notions and “pointers” for enhancing teaching.

Prelude to publishing articles on pedagogy in journals, say *PRIMUS – Problems, Resources, and Issues in Mathematics Undergraduate Studies* 😊.

## Uses of SIMIODE

Develop and maintain community in which groups of learners (class, project, study) and teachers (sharing, co-teach, co-builders) come together to learn and teach differential equations through modeling and technology throughout the entire process.

Return to origins of using differential equations to model change situations and enable understanding and intellectual growth.

Replace expensive text book with supportive community environment.

Demonstration of how the HUBZero environment can be used to support wide scale teaching and resource development for academic subject.

Some more examples. . . .

Build from first principles the heat and wave partial differential equations and then change tension in wave equation to tune string and play sound of solution in Mathematica's Play command; permit exploration of different string material; compare numerical with analytic solutions for physical (sound) and graphical attributes.

. . . . . and more in more detail.

# HANG TIME!

Let us consider the phenomena known as "hang time" in sports. Often the announcer in a televised football game will refer to the hang time for a punter's kick or a basketball announcer will say a player appears to "hang in the air."

What is that all about? Why would something appear to be so?

We consider a one dimensional representation of an object going up and coming down and ask what the notion of hang time might suggest in this context.

Suppose we consider a basketball player about to take a jump shot.

a) Estimate how high his hip rises from his standing position to the top of his jump.

Suppose we try to model the height of his hip and let us say  $y(t)$  is the height of his hip at time  $t$  seconds, where  $y(0) = 0$  is the starting height shifted to compensate for his real hip height.

b) Newton's Second Law of Motion says that the sum of the forces acting on a body are equal to the product of the body's mass and its acceleration,  $m \cdot y''(t)$ , it experiences can be computed by summing all the external forces acting on the body. Use that Law and the fact that the only force acting on the player's hip is due to gravity to build a differential equation for  $y(t)$ . What would be the initial conditions

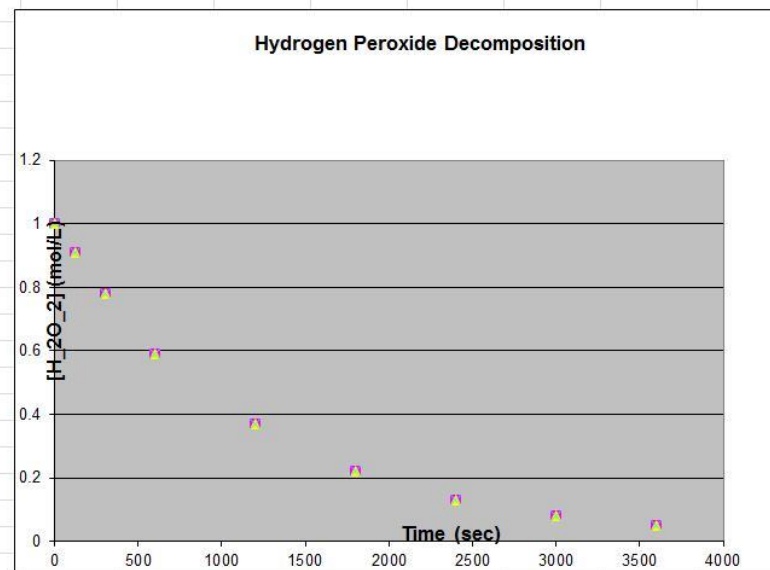
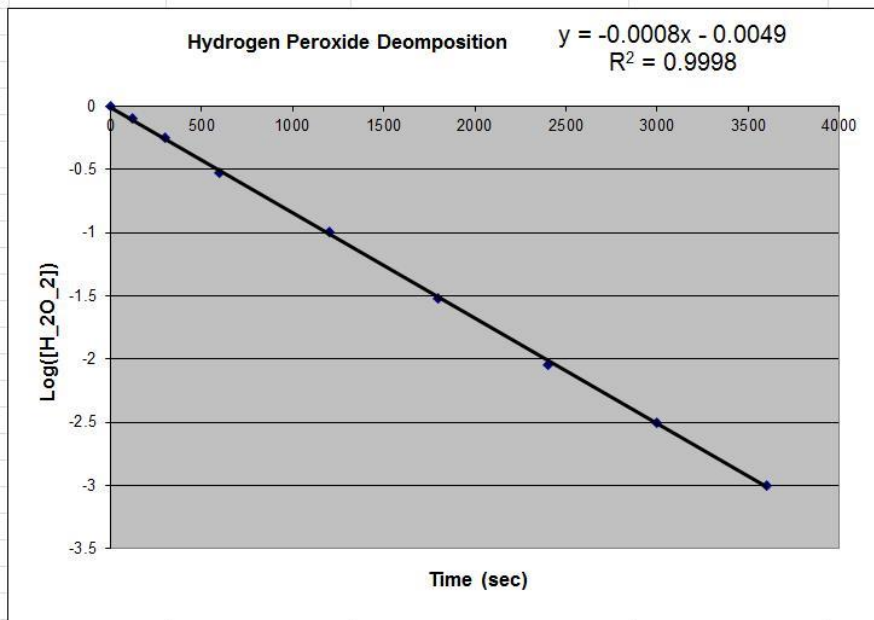


- c) Solve the differential equation with one of your initial conditions being unknown.  
Which condition is unknown?
- d) Based on your observations for (a) and your solution in (c) determine numerical values for your unknown initial condition.
- e) Now use your model with your initial condition to determine the times at which the player's hip is at 75% of its maximal height.
- f) Determine the entire length time of this player's jump.
- g) What percentage of time does the player's hip spend higher than 75% of its maximum height?
- h) Relate the finding in (g) to the appearance of hang time alluded to in the opening of this activity.

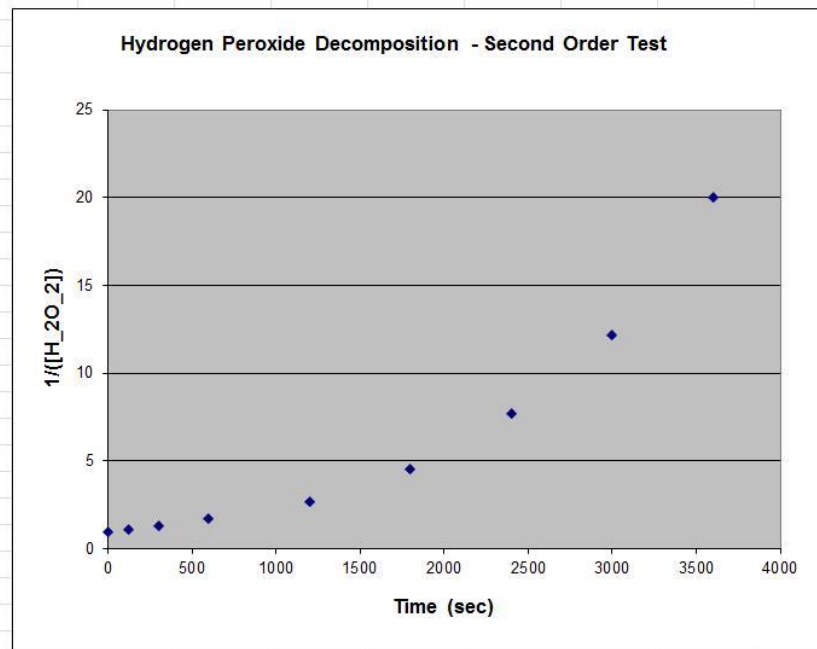
From Zumdahl, Steven S. Chemical Principles. 1992. Lexington MA: D.C. Heath.

Exercise 15 on p. 682.

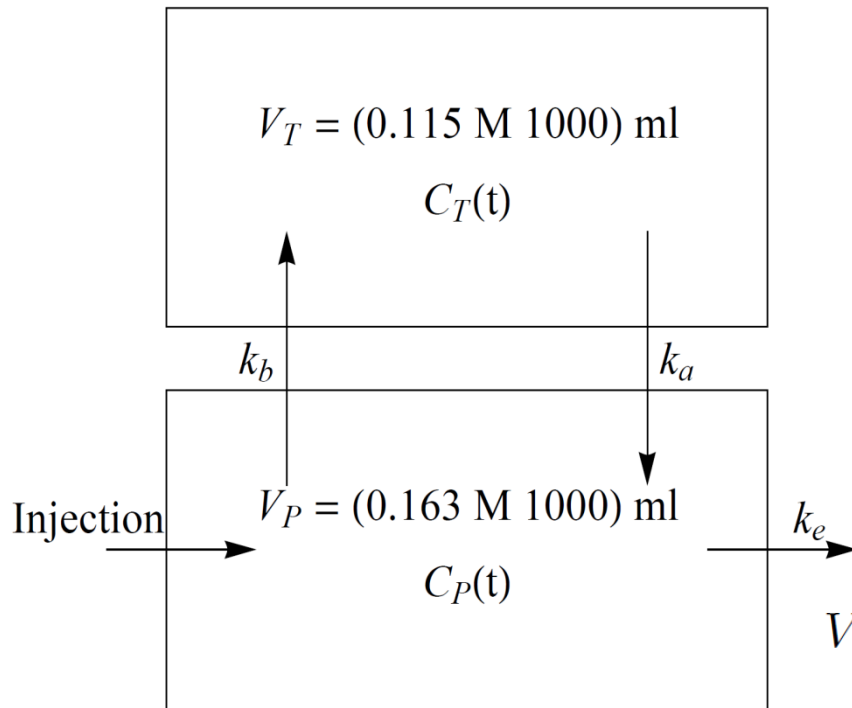
Time (sec)	[H <sub>2</sub> O <sub>2</sub> ] (mol/L)	LN ([H <sub>2</sub> O <sub>2</sub> ])	1/[H <sub>2</sub> O <sub>2</sub> ]	First Order Model
0	1	0	1	0.995111985
120	0.91	-0.094310679	1.098901099	0.904023431
300	0.78	-0.248461359	1.282051282	0.782782813
600	0.59	-0.527632742	1.694915254	0.61575877
1200	0.37	-0.994252273	2.702702703	0.3810213
1800	0.22	-1.514127733	4.545454545	0.235769652
2400	0.13	-2.040220829	7.692307692	0.145890345
3000	0.082	-2.501036032	12.19512195	0.090274523
3600	0.05	-2.995732274	20	0.055860375



Testing for second order we see that the reaction is not second order.



## Modeling Lysergic acid diethylamide (LSD) in the human body



$C_p(0) = 12.2699$ , for originally LSD was injected at a concentration of 2,000 ng per kg of body mass for each subject and so we have an initial concentration in the plasma of  $12.2699 = 2000M / .163M1000$  ng of LSD/kg of body mass.  $C_T(0) = 0$ .

$$V_P = \underbrace{(0.163M)}_{\text{kg of plasma}} \cdot \underbrace{(1)}_{\text{liter/kg}} \cdot \underbrace{(1000)}_{\text{ml/liter}} = \underbrace{(163M)}_{\text{ml of plasma}}$$

$$V_P C_P'(t) = k_a V_T C_T(t) - k_b V_P C_P(t) - k_e V_P C_P(t)$$

$$V_T C_T'(t) = k_b V_P C_P(t) - k_a V_T C_T(t).$$

	Time (hr)	0.833	0.25	0.5	1.0	2.0	4.0	8.0
Subject 1	Plasma Conc (ng/ml)	11.1	7.4	6.3	6.9	5.	3.1	0.8
	Perform Score (%)	73	60	35	50	48	73	97
Subject 2	Plasma Conc (ng/ml)	10.6	7.6	7.	4.8	2.8	2.5	2.
	Perform Score (%)	72	55	74	81	79	89	106
Subject 3	Plasma Conc (ng/ml)	8.7	6.7	5.9	4.3	4.4	—	0.3
	Perform Score (%)	60	23	6	0	27	69	81
Subject 4	Plasma Conc (ng/ml)	10.9	8.2	7.9	6.6	5.3	3.8	1.2
	Perform Score (%)	60	20	3	5	3	20	62
Subject 5	Plasma Conc (ng/ml)	6.4	6.3	5.1	4.3	3.4	1.9	0.7
	Perform Score (%)	78	65	27	30	35	43	51

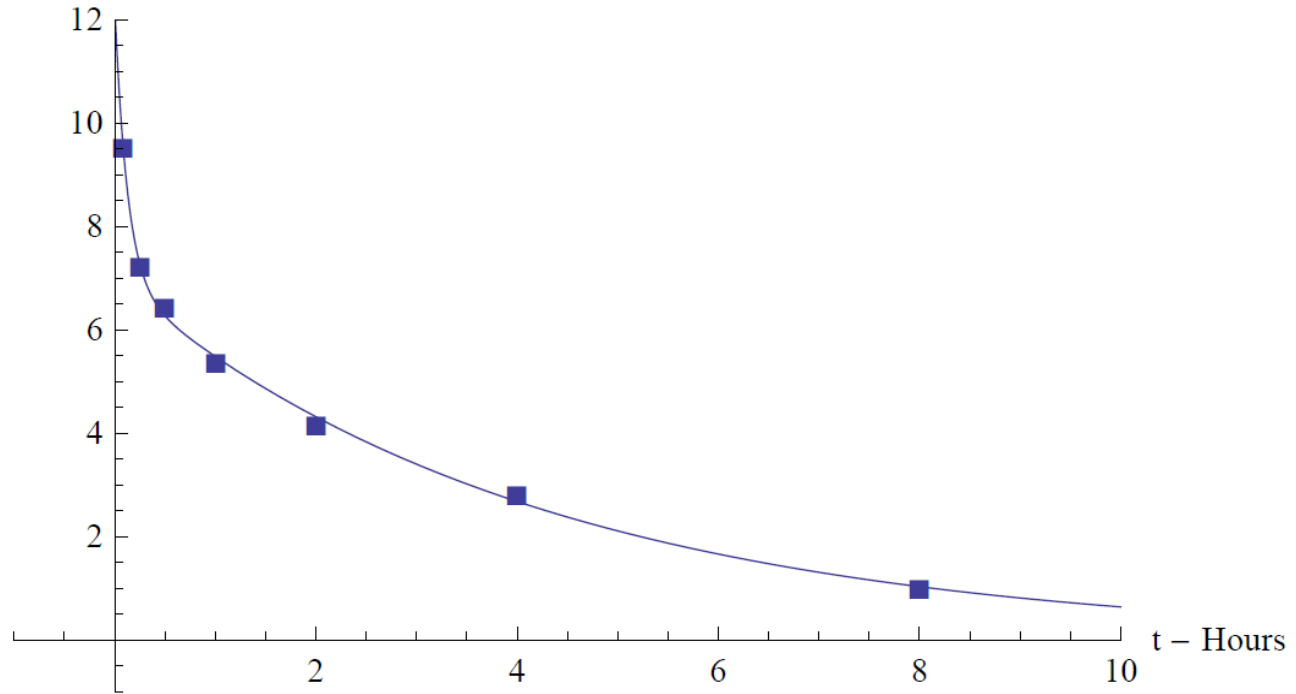
**Table 2.** Summary of data collected [1, 14] on 5 male volunteers who were administered LSD and then tested on performance (Perform Score (%)) on simple addition questions. Both performance Score and Plasma Concentrations of LSD were recorded at 5, 15, 30, 60, 120, 40, and 480 minutes after the initial infusion of LSD.

This is the formula for  $C_T(t)$ , the concentration of LSD in the tissues as a function of time  $t$ .

$$\left( -17.3913 e^{0.5 \left( -1. ka - 1. kb - 1. ke - 1. \sqrt{-4. kb ke + (1. ka + 1. kb + 1. ke)^2} \right) t} kb + 17.3913 e^{0.5 \left( -1. ka - 1. kb - 1. ke + \sqrt{-4. kb ke + (1. ka + 1. kb + 1. ke)^2} \right) t} kb \right) / \left( \sqrt{-4. kb ke + (1. ka + 1. kb + 1. ke)^2} \right)$$

$$SSE(k_a, k_b, k_e) = \sum_{i=1}^i (C_P(t_i) - O_i)^2 \quad (3)$$

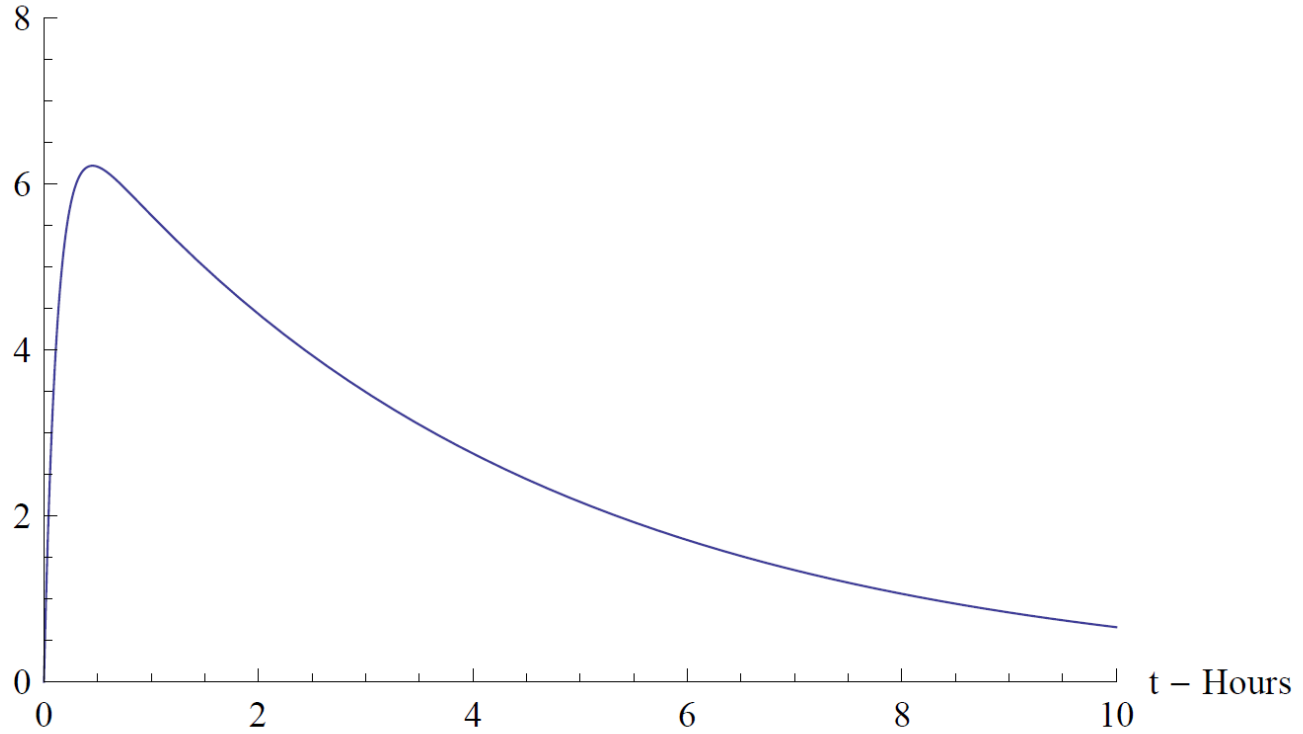
Plasma Conc. LSD 25 – ng/ml



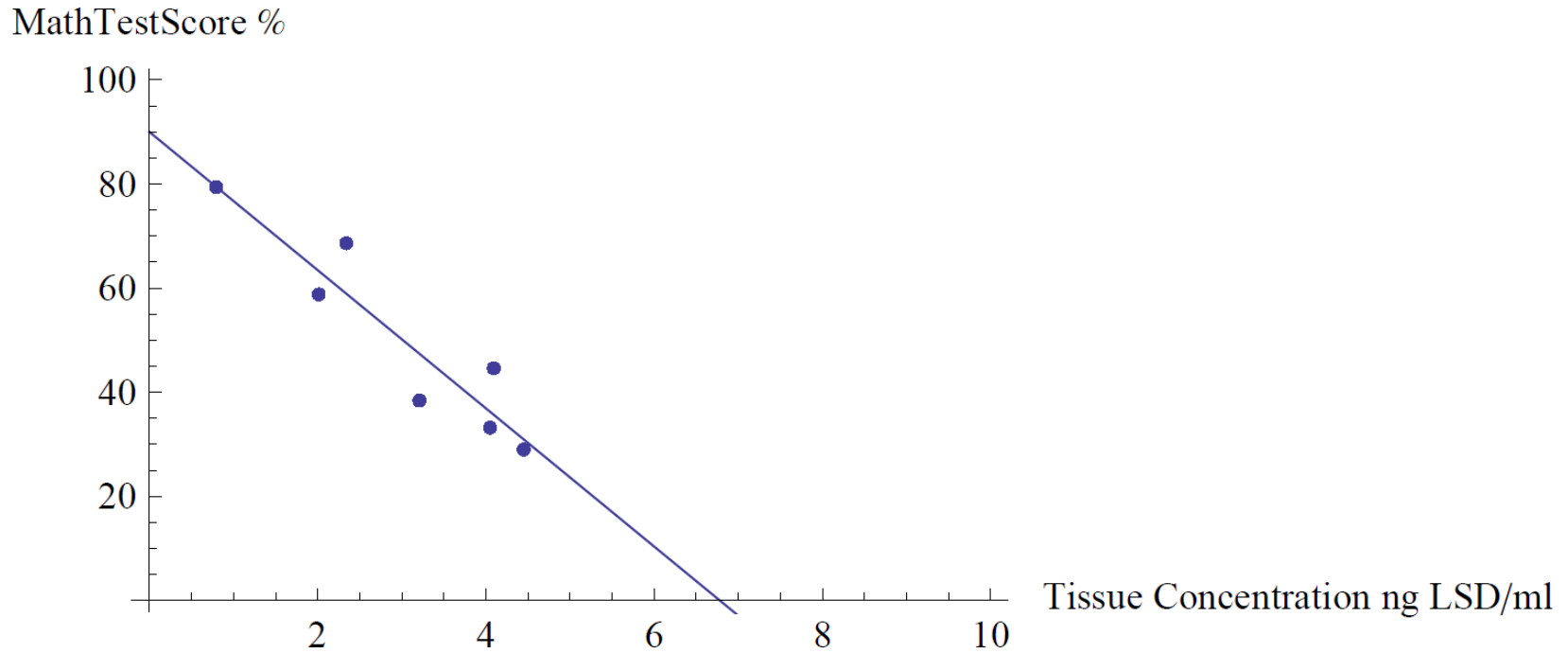
**Figure 2.** Plot of the observed values of the average concentration of LSD (ng/ml) (squares) and the model built from parameter ( $k_a$ ,  $k_b$ , and  $k_e$ ) estimates using the solution of the system of differential equations (2) and minimization of the sum of square error function (3).

$$C_T(t) = 0.128905 (55.419e^{-0.238492t} - 55.419e^{-7.99617t}) . \quad (5)$$

Tissue Conc. LSD 25 – ng/ml



**Figure 3.** Plot of the model of tissue concentration of LSD in ng/ml. This model is built from parameter ( $k_a$ ,  $k_b$ , and  $k_e$ ) estimates using the solution of the system of differential equations (2) and minimization of the sum of square error function (3).



**Figure 4.** Plot of Performance Score (%) ( $PS$ ) on the simple arithmetic problems vs. the model prediction of the concentration of LSD in ng/ml in the tissue compartment ( $CT$ ). Superimposed is the best line of regression whose equation is  $PS = 89.1729 - 9.32941CT$ . This means for every ng/ml increase in LSD in the tissue compartment the score drops a little over 9 points.



# Tuned Mass Dampers

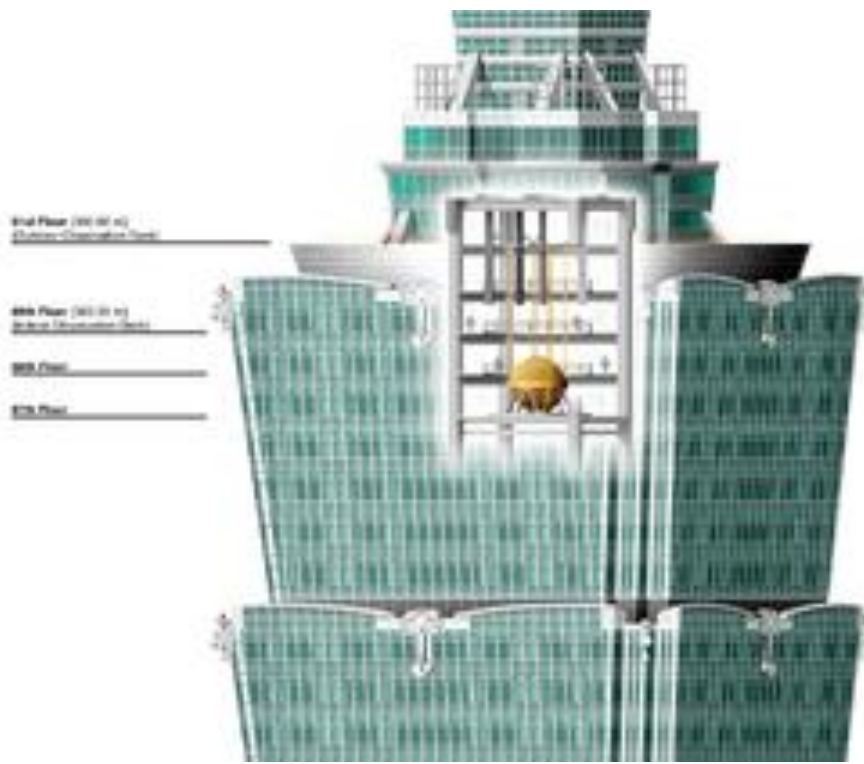
A Tuned Mass Damper (TMD) is a passive mechanical counterweight for a structure consisting of a moving mass (roughly 1–2% of the structure's mass) which is usually placed in the upper portion of the structure. The purpose of the TMD is to reduce the effects of motion caused by wind or earthquake.

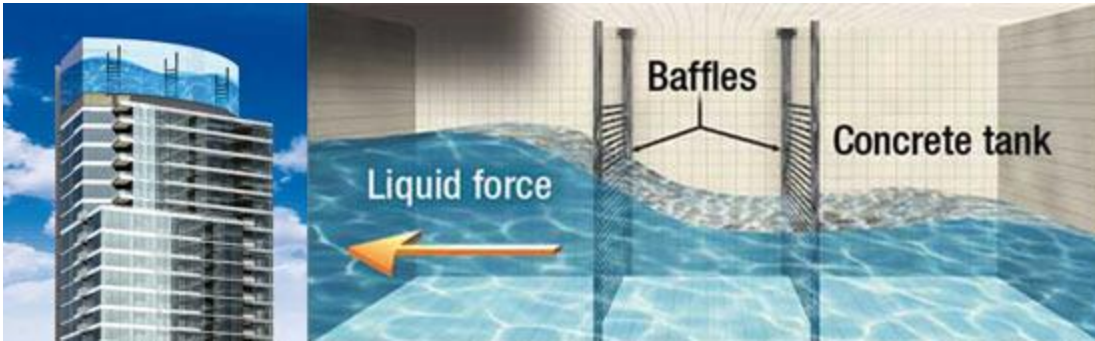
The first uses of TMDs in the United States for large structures was in the John Hancock Building in Boston in 1977 and City Corp Center in New York in 1978.

TMD's are used in many, many structures and devices, including buildings and bridges, electric razors, rotating tools, surgery table platforms, etc.

So what is a TMD? Well it is just a system of two second order differential equations And a physical device to implement the results.

TMD's can be in the form of sliding slabs (HUGE) of concrete and steel, sloshing tanks of water, pendula, and more.



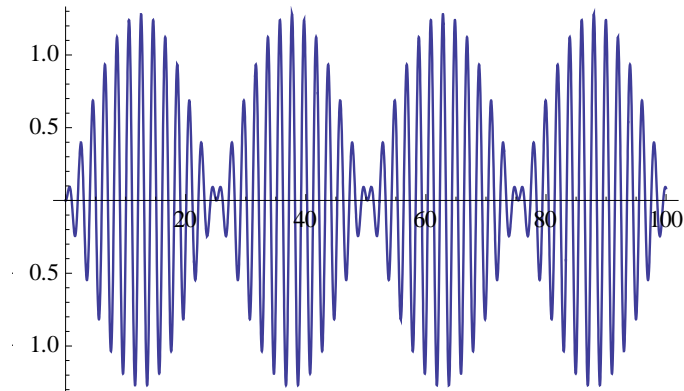


Harmonic Oscillator, e.g., building or bridge OR oboe reed.

$$1 y''(t) + \omega_0^2 y(t) = \cos(\omega t), \quad y(0) = y_0, \quad y'(0) = v_0$$

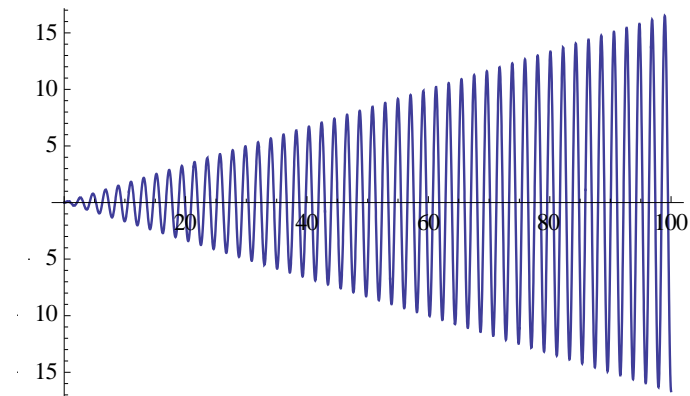
When  $\omega_0$  and  $\omega$  are close we get beats,

e.g.  $\omega_0 = 3.0$  and  $\omega = 3.25$



When  $\omega_0$  and  $\omega$  are equal we get resonance,

e.g.  $\omega_0 = 3.0$  and  $\omega = 3.0$



Thus if in 1  $y''(t) + \omega_0^2 y(t) = \cos(\omega t)$ ,  $y(0) = y_0$ ,  $y'(0) = v_0$   
We have  $\omega_0 = \omega$ , i. e. the driver frequency is the same as the natural frequency of the system/structure, then we can have resonance and the system./structure can have giant and dangerous oscillations. For a building this can come from wind or earthquakes. What can we do to mitigate this danger?

Differential equations come to the rescue!

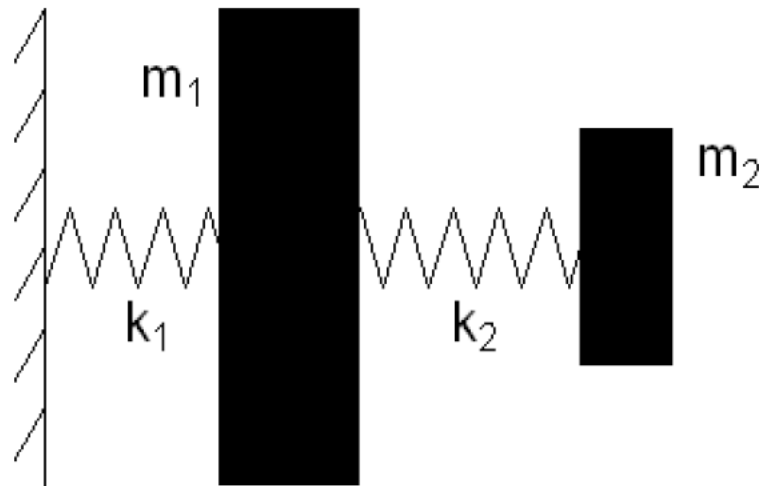
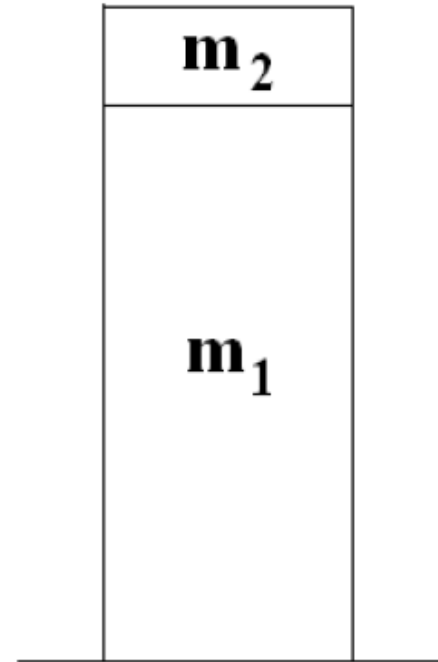
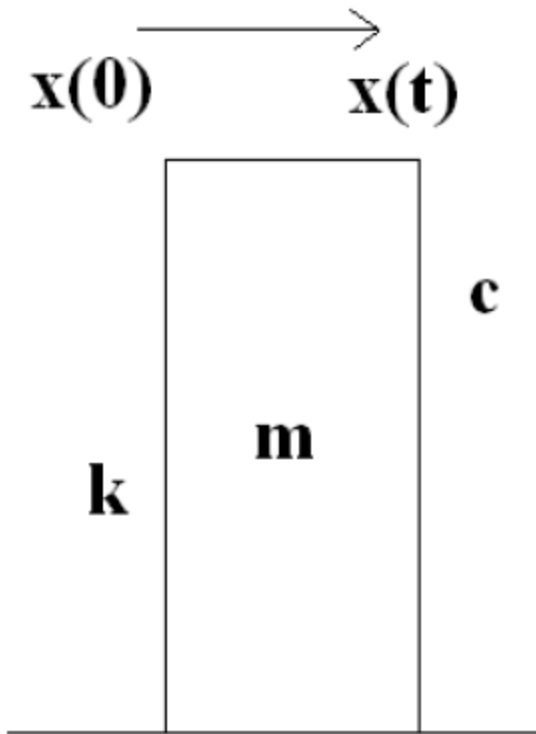


Figure 6: Horizontal depiction of two mass spring system (no damping on either mass) with smaller mass  $m_2$  serving as Tuned Mass Damper.



Depiction of two masses for a structure – smaller mass on top.  
K (spring constant) is called stiffness and c is still called damping coefficient.

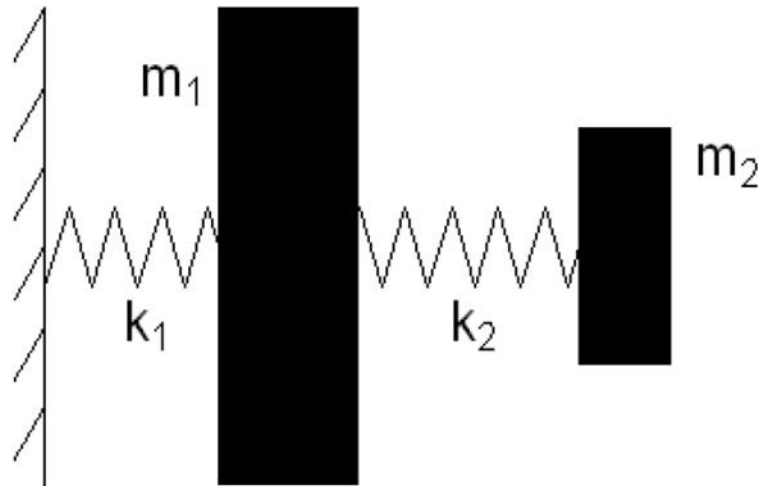


Figure 6: Horizontal depiction of two mass spring system (no damping on either mass) with smaller mass  $m_2$  serving as Tuned Mass Damper.

$$\begin{aligned}
 m_1 x_1''(t) + (k_1 + k_2)x_1 - k_2 x_2 &= F(t) \\
 m_2 x_2''(t) + -k_2 x_1 + k_2 x_2 &= 0.
 \end{aligned}$$

In the simplified case where there is no damping (i.e.  $c = 0$ ) then we might seek to control for resonance by adding a second “tuned” mass.

We now “drive” the larger mass with a force. . wind, earthquake

$$m_1 x_1''(t) + k_1 x_1(t) + k_2 x_1(t) - k_2 x_2(t) = F_0 \cos(\omega t)$$

$$m_2 x_2''(t) - k_2 x_1(t) + k_2 x_2(t) = 0 .$$

If no second mass then we could have resonance ( $c=0$ ) or high amplitude.

So we add a second smaller mass and we “tune” it so that its system has the same frequency as that of the larger mass.

The natural frequency of our structure is  $\omega = \sqrt{\frac{k_1}{m_1}}$  and of the added mass the frequency is  $\sqrt{\frac{k_2}{m_2}}$ . Consider  $k_1 = 90$  and  $m_1 = 10$  with  $k_2 = 0.90$  and  $m_2 = 0.10$  .

Then both spring systems have the same frequency, namely  $\omega = \sqrt{\frac{90}{10}} = \sqrt{\frac{0.9}{0.1}} = 3$ .



Now, by examining the solution of our system

$$\text{mass } m_1 : \quad -\cos(t\omega) + \frac{909}{10}(a \cos(t\omega) + b \sin(t\omega)) - \frac{9}{10}(c \cos(t\omega) + d \sin(t\omega)) \\ + 10(-a \cos(t\omega)\omega^2 - b \sin(t\omega)\omega^2) = 0$$

$$\text{mass } m_2 : \quad -\frac{9}{10}(a \cos(t\omega) + b \sin(t\omega)) + \frac{9}{10}(c \cos(t\omega) + d \sin(t\omega)) \\ + \frac{1}{10}(-c \cos(t\omega)\omega^2 - d \sin(t\omega)\omega^2) = 0.$$

we can combine terms, simplify, and determine that the amplitude of the resulting oscillation of mass is

$$\text{amp}(\omega) = 10 \left| \frac{\omega^2 - 9}{100\omega^4 - 1809\omega^2 + 8100} \right|$$

A picture is worth . . . . How many words . . . or equations?

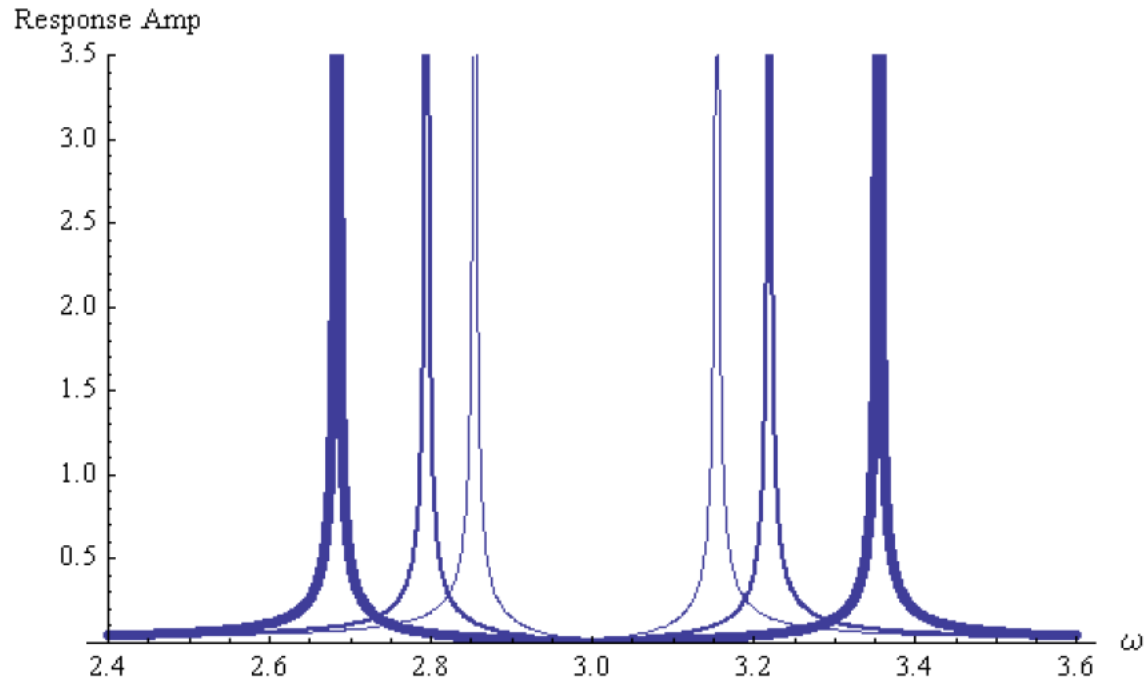


Figure 9: In case neither mass  $m_1$  nor  $m_2$  has damping, i.e.  $c_1 = c_2 = 0$ , here are plots of the response amplitudes of the primary system mass,  $m_1$ , as a function of the driver frequency  $\omega$ . We see in the three plots that as the ratio  $\frac{m_2}{m_1}$  of added secondary mass  $m_2$  as a percentage of the primary mass,  $m_1$ , goes from 1%, to 2%, to 5%, (corresponding to the thin, thick, and thicker plots, respectively) the frequency region of low responses to the driver frequency surrounding the natural frequency of the structure expands.

So if we are willing to add more and more mass we get a wider “safe” region.

Incidentally, if we go back to just our one mass system

$$m \cdot y''(t) + c \cdot y'(t) + k \cdot y(t) = f(t), \quad y(0) = y_0, \quad y'(0) = v_0$$

with a driving force  $f(t) = F_0 \cos(\omega t)$

Here is the steady state (non-homogeneous) solution for Equation (4) with  $c \neq 0$ :

$$\text{ss}(t) = -\frac{F_0 m \cos(\omega t) \omega^2 + F_0 k \cos(\omega t)}{m^2 \omega^4 + c^2 \omega^2 - 2km\omega^2 + k^2} + \frac{cF_0 \sin(\omega t) \omega}{m^2 \omega^4 + c^2 \omega^2 - 2km\omega^2 + k^2}.$$

The amplitude of the steady state solution as a function of input frequency,  $\omega$ , is given in Equation (5).

$$\text{amp}(\omega) = F_0 \sqrt{\frac{1}{m^2 \omega^4 + c^2 \omega^2 - 2km\omega^2 + k^2}} = F_0 \sqrt{\frac{1}{c^2 \omega^2 + (m\omega^2 - k)^2}}. \quad (5)$$

How big a response (i.e. how big can  $\text{amp}(\omega)$  really be?

We seek the maximum frequency response of  $\text{amp}(\omega)$ . Again, a picture . . . .

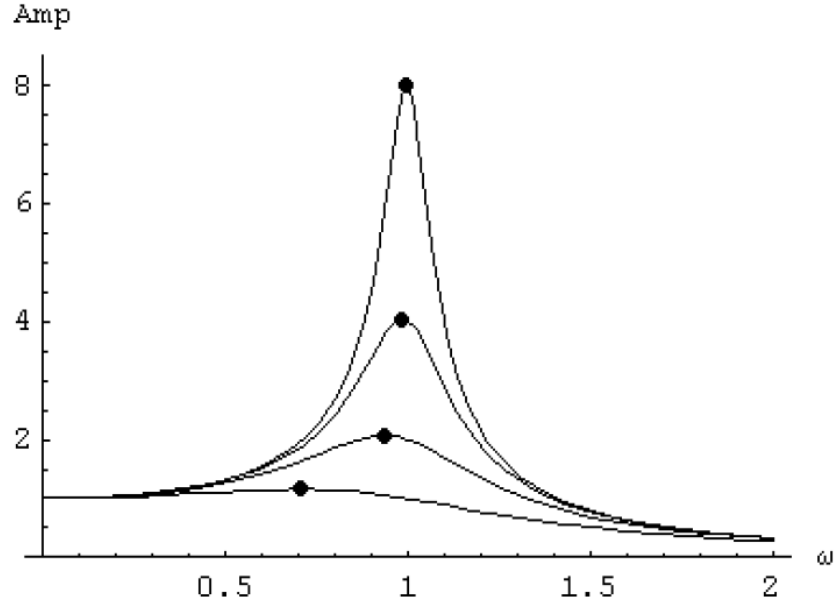
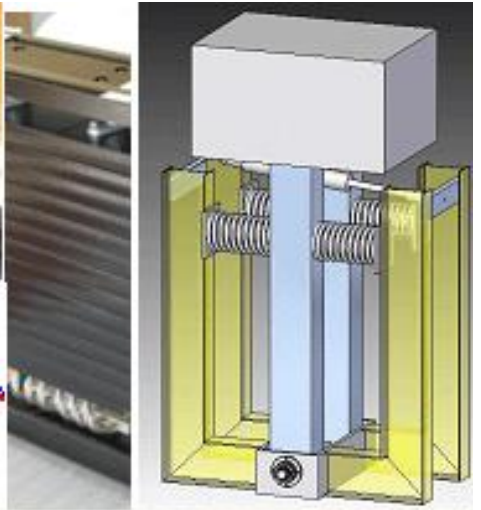
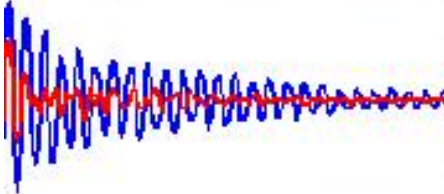
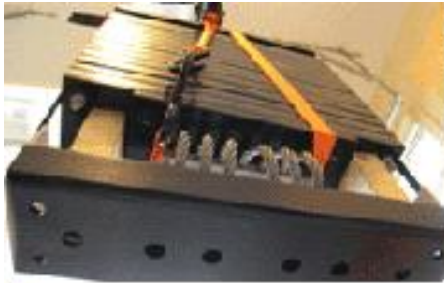
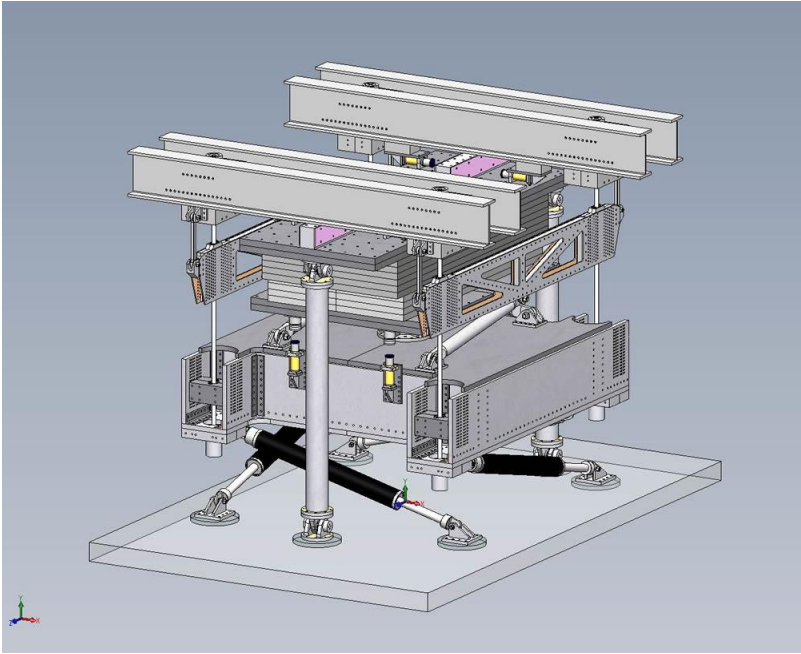


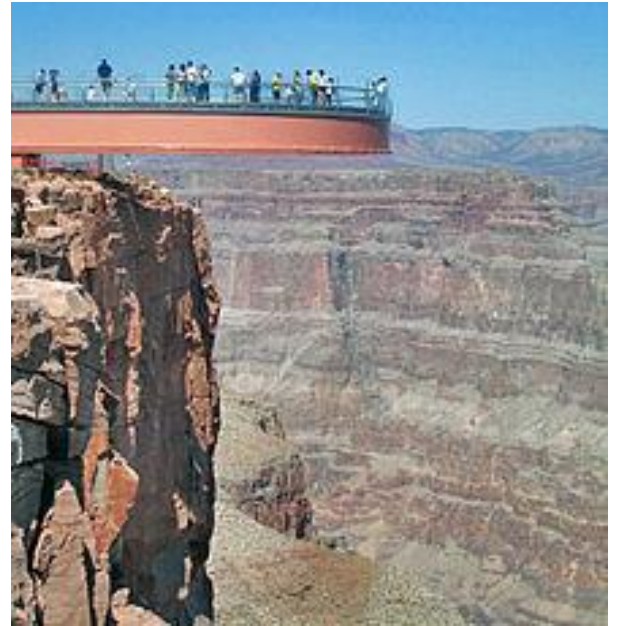
Figure 3: Plot of peak frequency response, both input frequency,  $\omega$ , and amplitude, of steady state solution for  $m = 1$ ,  $k = 1$ ,  $\omega_0 = 1$ ,  $F_0 = 1$ , for  $c = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$  where peak is highest for lowest value of  $c$ , i.e.  $\omega_{\max} \rightarrow \omega_0$  as  $c$  decreases. These values appear in Table 1.

$c$ value	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
Peak response frequency, $\omega_{\max}$	0.707107	0.935414	0.984251	0.996086
Maximum response amplitude	1.1547	2.06559	4.03162	8.01567

Table 1. Peak frequency response for Equation (4) with  $m = 1$ ,  $k = 1$ ,  $F_0 = 1$ , for  $c = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ .









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