Managing Project Logistics in Teaching Differential Equations

Corban Harwood

George Fox University
2018 Joint Mathematics Meeting

January 10, 2018
1. Pedagogical Foundation
2. Case Study
3. Assessment
Pedagogical Foundation

Goals for Project-Learning Activities

1. Accountability
   - Track/grade individual contribution in posting and presenting

2. Communication
   - Oral presentations

3. Interdisciplinary
   - Applications require investigation beyond course topics

4. Data Integration
   - Generate/access data to analyze
   - Preprocessing
   - Error analysis with model solution

5. New Problems
   - New context, new format for problem statement
Example Project: 2nd order equations

Designing Shock Absorbers for NASCAR

- Search for quantitative and qualitative data on NASCAR vehicle and driving conditions
  - Smoothness of NASCAR track
  - Rigidity of steering control needed for high speeds
- Analyze model for patterns (DFIELD applet, WolframAlpha)
- General Spring-Mass model
  - Investigate optimal parameters and evaluate model
- Cap off 2nd order differential equations chapter
1. Generate data motivated by reality to introduce project
2. Analyze data to develop model
3. Solve and evaluate model
4. Segue into topics motivated by the project
Example Project: 1st order equations

Simulating the Spread of the Common Cold (Link)

- Generate data with hands-on simulation
- Analyze data for patterns (DFIELD applet)
- Develop, analyze, and solve ODE models with parameters
- Estimate parameters and evaluate model
- Segue into topics motivated by the project
Attributes of Project-Based Learning

1. Open-ended driving question
   - Creates a need to know
   - Inquiry to learn something new

2. Critical thinking, Collaboration, and Communication

3. Student voice and choice

4. Incorporates feedback and revision

5. Publicly presented
Example Week 13 Project

Developing a Model for Two-Species Interactions

- Search for quantitative and qualitative data on chosen species
- Analyze data for patterns (PPLANE applet)
- Develop, analyze, and solve ODE models with parameters
- Estimate parameters and evaluate model
- Segue into topics motivated by the project
Project Implementation Best Practices

- Start small
  - One project in one class
  - Short class-wide project
- Minimize additional workload
  - Reduce evaluation/grading
  - Model group work
  - Course management system
  - Use technology on hand
- Reinforce project purpose
  - Continuity between project and course content
  - Fair division of group work
Resources for Projects

- **Repositories**
  - SIMiode: Differential Equations (http://www.simiode.org)
  - Project MOSAIC: Calculus (mosaic-web.org)

- **Journals**
  - PRIMUS journal: Teaching Resources
  - MAA Journals: Expository Mathematics and Teaching Resources

- **Public Data**
  - NOAA (CO₂ Data)
  - Economic Database (GDP, ect)
Case Study

Course Structure

MATH 311: Differential Equations with Linear Algebra (75 minutes, 3 days)
Three 2-Week Projects

- Groups assigned with some choice
- Open-ended problems/problem choice
- Write-up: Series of linked wiki pages
  - Describe the phenomena modeled
  - Derive the model system (assumptions, variables, and initial conditions)
  - Analyze model and solution
- Short oral report in class recorded as a screencast
Week 1 Project

A) Record your simulations of Brandt Outbreak Model (Section A) in this linked spreadsheet.

B) Derive/choose a differential equation model (with unknown parameters) and find its general solution.

C) For each simulation, estimate the fitting parameter (growth rate $a$) by graphing the general solution with each data set. Make sure your best fitting graph is shown in your group’s tab on the spreadsheet.

D) As a class on Monday, we will combine all the graphs and analyze the spread of the parameter values and work on the wiki write up. Afterwards, you will post a summary on the wiki:

* Summarize why your model could possibly fit. Explain in detail the main steps needed in solving your model equation. Show the model and solution.
* Was it a good model (could you fit the data well)?
* Was your parameter value(s) close to the average?
* How could this model be used?
* How could this model be improved?
Week 7 Project

A) Imagine that you are working with a Nascar team to design coilover shock absorbers for a race car. Given the minimum allowed car+driver weight, you have modeled each shock as a spring mass system with a mass of 175 kg with spring constant of 30,000 N/m. Choose values for the damping coefficient of the inner shock fluid to design systems of the following damping types. Which type do you think will provide the best racing experience for the driver? (Overdamped, Critically Damped, Underdamped, Undamped)

B) Using your value of b from your chosen best behavior type (above), consider the scenario where the driver has been slightly bumped by another car and course-corrects back and forth until she straightens out. You model this course-correction as forced oscillations of $f(t) = F \sin(\omega t)$. What frequency (in Hz) is most dangerous for her course-corrections? Should she course-correct more or less frequently to best minimize the amplitude?
Project Steps: Write up the main steps from your work in your wiki page
1. Choose your group’s topic based on interacting species, products, groups, etc. and choose valid signs for a, b, c, d in the general system to model the interactions and preserve a coexistence equilibrium point (both nonzero).
2. Choose reasonable integer values for a, b, c, d to represent the interactions (normally they would be fractional percentages, but integers are easier to solve with).
3. Use Laplace transforms to solve the linearized system near the coexistence equilibrium point (including the shift by the eq. pt. \( x^* = (x^*, y^*) \)) as \( x' = A(x - x^*) \).
4. Graph the system in PPLANE for your parameters and one or more with slightly different parameter values to show how it changes.
5. Find a short video clip or series of images describing your topic to share.
### Assessment

**Evaluation through Feedback**

- **Student feedback in Differential Equations**

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(x: 2-25%, X: 25-100% of written comments about projects)
Likert scales encourage focused feedback: “Projects for this course...
(1) Added new perspectives on the content covered in homework”
(2) Helped me understand and engage with course concepts more fully”

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Future Goals

- Improve rhythm and balance
- Develop students’ thirst for inquiry
- Improve inter-group communication
- Instill the cyclic nature of the modeling process
Thank You


