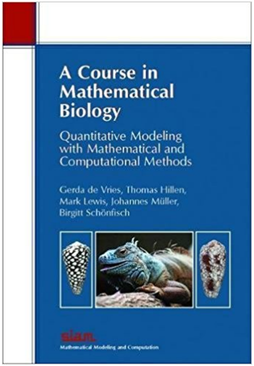
Qualitative Analysis of Differential Equations and the Zombie Apocalypse in a Mathematical Biology Course

Christina Alvey
Mount Saint Mary College

January 10, 2018 – JMM

MTH 3850: Special Topics: Mathematical Biology

- This course was NOT Differential Equations.
- Prerequisite: Calculus II
- Textbook: De Vries, G., et al. (2006). *A course in mathematical biology: quantitative modeling with mathematical and computational methods*. (Vol. 12) Siam.



Some Covered Topics... and all before Spring Break!

- Discrete-Time Models
  - logistic growth
  - cobweb analysis
  - fixed points, solution plots, stability, and phase lines for one difference equation
  - fixed points, stability, and phase portraits for systems of two difference equations
- Continuous-Time Models
  - intro to differential equations
  - equilibria, phase-line analysis, stability theorem, and solution plots for one ODE
  - vector fields
  - predator/prey
  - equilibria, stability, and nullclines for systems of two ODEs
  - bifurcation diagrams

And finally... SIR models!

In: Infectious Disease Modelling Research Progress ISBN 978-1-60741-347-9
Editors: J.M. Tchuenche and C. Chiyaka, pp. 133-150

When Zombies Attack!: Mathematical Modelling of an Outbreak of Zombie Infection

Philip Munz[1,*], *Ioan Hudea*[1,†], *Joe Imad*[2,‡], *Robert J. Smith?*[3,§]

Abstract

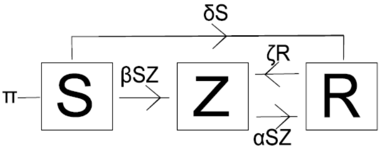
Zombies are a popular figure in pop culture/entertainment and they are usually portrayed as being brought about through an outbreak or epidemic. Consequently, we model a zombie attack, using biological assumptions based on popular zombie movies. We introduce a basic model for zombie infection, determine equilibria and their stability, and illustrate the outcome with numerical solutions. We then refine the model to introduce a latent period of zombification, whereby humans are infected, but not infectious, before becoming undead. We then modify the model to include the effects of possible quarantine or a cure. Finally, we examine the impact of regular, impulsive reductions in the number of zombies and derive conditions under which eradication can occur. We show that only quick, aggressive attacks can stave off the doomsday scenario: the collapse of society as zombies overtake us all.

What should we include in a human/zombie model?

Some Student Responses

- how fast zombies walk
- zombies biting humans
- humans killing off zombies
- location of zombie attack
- how many zombies have already been created
- can zombies rise from the dead?
- can zombies swim?

The first zombie model:



$$
\begin{aligned}
S' &= \pi - \beta SZ - \delta S \\
Z' &= \beta SZ + \zeta R - \alpha SZ \\
R' &= \delta S + \alpha SZ - \zeta R
\end{aligned}
$$

- constructed compartmental model ourselves
- came up with equations based on compartmental diagram
- found equilibria and discussed disease-free vs endemic
- determined stability

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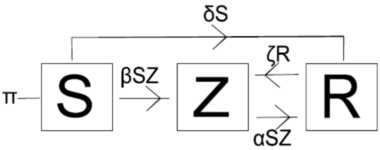


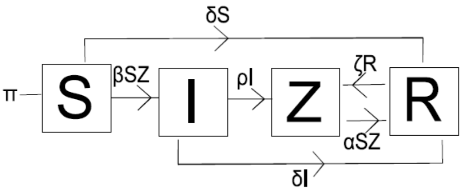
$$\begin{aligned} S' &= \pi - \beta SZ - \delta S \\ Z' &= \beta SZ + \zeta R - \alpha SZ \\ R' &= \delta S + \alpha SZ - \zeta R \end{aligned}$$

- introduced R_0, what it means, and how to find it

$$R_0 = \underbrace{\beta}_{\text{transmission rate}} \cdot \underbrace{\frac{1}{\alpha}}_{\text{zombie lifetime}}$$

Other zombie models:

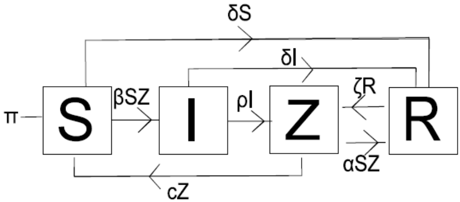
Assumption: Humans are turned into zombies by getting bitten, but this transformation might not take place immediately.



$$R_0 = \underbrace{\beta}_{\text{transmission rate}} \cdot \underbrace{\frac{\rho}{\delta + \rho}}_{\text{probability that infected humans become zombies}} \cdot \underbrace{\frac{1}{\alpha}}_{\text{zombie lifetime}}$$

Other zombie models:

Assumption: There is a cure for zombieism.



$$R_0 = \underbrace{\beta}_{\text{transmission rate}} \cdot \underbrace{\frac{\rho}{\delta + \rho}}_{\text{probability that infected humans become zombies}} \cdot \underbrace{\frac{1}{\alpha + c}}_{\text{zombie lifetime}}$$

Student comments about zombie part of class...

- "Really interesting class, it was fun learning about zombies"
- "From the description I thought the class would be more fun and more focused on zombies and stuff like that but it feels like we just glanced over it."

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- "Really interesting class, it was fun learning about zombies"
- "From the description I thought the class would be more fun and more focused on zombies and stuff like that but it feels like we just glanced over it."
- But... the other half of the class was bored with zombies and so I decided to talk about other things...

Greyscale Project

- When an individual is infected with Greyscale, there are no signs of the disease for 180 days after which grey, scaly skin appears first at the site of contact, then spreads throughout the body.
- There is no cure, but treatment can stop the spread of disease. Assume the treatment rate is 0.0005 per day.
- Those not treated by a doctor are sometimes exiled from their homes (at a rate of 0.008 per day), but otherwise live a normal life.
- At the beginning of the year, there are no infected people in the city, which has a population of 20,000. On the first day of the year, a ship arrives with 100 men who are afflicted with an advanced form of Greyscale.They will spread the disease at a rate of 0.012 per day.
- Assume the lifespan of an average individual at that time was 50 years and that the population size was constant over time.

Other topics:

- more SIR-type models involving real diseases
  - influenza
  - varicella
  - ebola
  - SARS
- sensitivity analysis
- herd immunity
- Leslie matrices
- final projects

If I teach this class again...

- Continue to use zombies as an introduction to SIR models.

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- Continue to use zombies as an introduction to SIR models.
- Emphasize zombies a little less in the course description.
- Have students create their own zombie model–or if they're sick of zombies, to make up their own disease!

Thank You!