**Executive Summary Team 11-09**

**Statement of Problem:** Design an underwater table tennis game.

We use a steel ball of radius 1.2 cm, paddle strung like a tennis racquet about the size of a table tennis paddle, and a concrete table 274 cm long, 152.5 cm wide, 152 cm high. Our centered net is 45.25 cm high. Our table is regulation size while our net is three times as high as regulation size. The table is at the bottom of a flat pool in 2.5 m (250 cm) of fresh water. The racquet must be porous to permit imparting high initial velocity to the ball by the hitter in order to clear the high net and get near the table's edge on the opponent's side of the table. We have three rules.

(1) If the ball breaches the surface of the water the striker loses the point.
(2) The ball must strike the opponent's side of the table first every play.
(3) After the ball strikes the opponent's side of the table once it must bounce off the table surface.

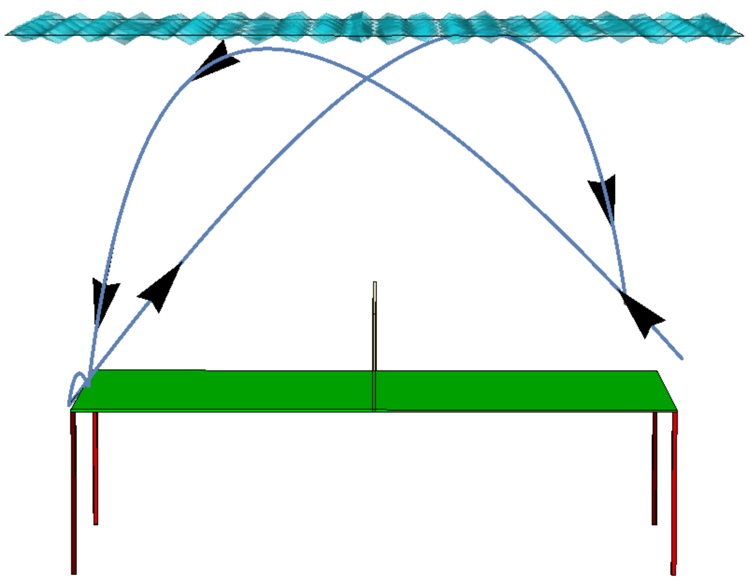
From Newton's Second Law of Motion we build a governing equation for the velocity of our ball in cm/sec. We use a Free Body Diagram to sum external forces (1) acting on the ball. The nature of the forces are identified above terms.

$$\rho_B * v * \mathbf{V}'(t) = \overbrace{-\,\alpha \boldsymbol{V}(t)}^{\text{resistance}} \; \overbrace{-\,\rho_B * v * g\,\mathbf{(0,0,1)}}^{\text{gravity}} \; \overbrace{+\,\rho_{\text{Water}} * v * g\,\mathbf{(0,0,1)}}^{\text{buoyancy}} \qquad (1)$$

**V**(t) is the velocity of the ball in cm/s, $\rho_B$ = 7.6 g/cm$^3$ is the density of the stainless steel ball of radius $r$ = 1.2 cm we use; $v$ is the volume of the ball in cm$^3$; $\alpha$ = 119.86 dynes/(cm/sec) is the resistance coefficient to motion of the ball; $\rho_{\text{Water}}$ = 1 g/cm$^3$ is the density of fresh water in which the game is played; and $g$ = 980 cm/s$^2$ is the acceleration due to gravity. We estimated $\alpha$ by collecting data on the fall of a hard rubber ball of same radius (but less dense) as steel ball in our final model, in a graduated cylinder of water. The coefficient of resistance of the steel ball bouncing on the concrete table is e = 0.597 where $v_{up} = e^2 * v_{down}$ relates the velocity of the ball coming down, $v_{down}$, to the velocity of the ball going up, $v_{up}$.

We solve (1) for velocity and use the definition of velocity to create a differential equation for position and then solve for position. We tried many initial velocities and positions for service and found a number of reasonable return velocities for each player who hits the ball from below the surface of the table after it falls there from the bounce.

In traditional table tennis the goal is to slam the ball horizontally and low across the net. However, in our game slamming the ball upward and over the net is important, landing the ball on the opponent's side of the table, and bouncing off the table to permit the opponent to get a clear shot to hit the ball over the high net and locate the landing near the edge of the opponent's side of the table. However, the forward motion of the ball is slowed so rapidly due to resistance of water that the ball essentially goes until it stops horizontal motion and then falls to the table with little height to the bounce. Thus, we need the ball to fall near the edge and bounce off the table so the opponent can get a good hit on the ball to properly return the ball.



The "drop" of the steel ball once the apex is reached is due to loss of Kinetic Energy, i.e. reduced horizontal velocity because of the resistance due to water. This is in line with observations of projectile motion under water.

We give an example of the trajectory of a typical volley from initial position on the right (serve), followed by a return of service from the left. On the left side of the table we see the ball bounce off the table and then drop below the table to permit a good return. The server imparts a velocity of (-648.2, 0, 737.6) cm/s while the return of service by the opponent has a velocity of (614.7, 0, 838.2) cm/s.

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