Successful Modeling Motivation for Beam Equations.

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Introduction

- Boundary Value Problem - Beam Equation
- Solution Techniques Motivated by Data Collection and Analysis
- SIMIODE
- Classroom Organization
- Student Work
- Final Remarks
Hanging Beam Derivation

✔ Curvature \( k = \frac{\frac{d^2y}{dx^2}}{(1 + (\frac{dy}{dx})^2)^{3/2}} \)

✔ Small Deflection \( \Rightarrow \frac{dy}{dx} \approx 0 \Rightarrow k = \frac{d^2y}{dx^2} \)

✔ Bending Moment related to Load per Unit Length: \( \frac{d^2M}{dx^2} = w(x) \)
Hanging Beam Derivation

\( \frac{d^2 M}{dx^2} = w(x) \), Where \( M(x) = EI k \)

\( M = \) Bending Moment, \( E = \) Young’s Modulus, \( I = \) Moment of Inertia, \( w = \) weight, \( k=\) curvature, \( EI = \) Flexural Rigidity

\( \text{Governing Equation: } \frac{d^2 M}{dx^2} = w \Rightarrow \frac{d^2 EI k}{dx^2} = w \Rightarrow EI y^{(4)} = w \)
Alternative Content Motivation

✓ $E I y^{(4)} = w$  Where does this come from? Why are there differential equations?

✓ Contrive Data Collection Exercise

✓ Data Analysis Leads to Topic of Interest

✓ Student Discovery Instead of Instructor Dissemination
Ulterior Motives

✔ Not Just How, But Why.

✔ Solution Techniques as Part of a Process

✔ Scientific Method

✔ Reduced Time-frame Requires Contrivances
✔ Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

✔ Use Modeling and Technology Upfront

✔ Website: Many Pre-Built Modeling Scenarios, Room for More

✔ Brian Winkel, Director SIMIODE
SIMIODE Developer Workshop

✔ Invited to participate in a SIMIODE Developer Workshop in Helena MT

✔ 12 Participants worked for seven days on Scenario Creation.

✔ Conceive, Experiment, Refine, Write-up a Student and Teacher Version

✔ Mass Spring and Two-Mass Spring Systems
Specific Challenges

✔ Oregon Institute of Technology: Engineering and Specific Health-Science Degrees

✔ Required Content and Limited Time: Three Days

✔ Many Civil and Mechanical Engineering Students

✔ Hanging Beam and Buckling Column
Class Preparation

✔ Physically Perform with a Colleague ahead of time

✔ Written and Verbal Advance Notice, Formation of Student Teams

✔ Set-up: A Station for Each Team

✔ Work Together Outside of Class

✔ Multiple Deadlines
Three Hanging-Beam Scenarios:

- Pinned-Pinned
- Cantilever
- Embedded-Embedded
Data Collection Details

- Prescribed Data Collection
- Two Tables, Plastic Beam, Large Sheet of Paper, Ruler, Calculus Texts
Leading Questions

✔ Scatter Plot: Lowest Degree Polynomial Fit?

✔ Single Degree (fit) for all Three?

✔ What Differential Equation might an n’th Degree Polynomial be the Solution of?

✔ Speculation: \( \frac{d^n y}{dx^n} = k \)
Boundary Conditions?

✔ Given $\frac{d^n y}{dx^n} = k$, How Many Constants?

✔ Embedded-Embedded First! What is the value of the Derivative at each end?

✔ Pinned-Pinned: Are there Inflection Points? What do we know about Derivatives at an Inflection Point?

✔ Cantilever: Embedded End and no Curvature at Free End give Three Conditions. Give Students $y'''(L) = 0$. 
Expectations

✔ Write a Self-Contained, Well Written Report. Provide a Possible Template.

✔ Must Include: Data for all Three Scenarios
  Model(s) used to Fit the Data
  Differential Equations and Associated Solutions
  Graphs that Compare Data to Model(s) and Solutions
  Error Analysis

✔ Provide help Implementing a Plan but No Help with Planning.
Most Teams found 6th Degree Polynomial to be the "Best Fit" in Excel

Typical Imposed Condition: \( y'(L/2) = 0 \)

All Teams Submitted a Thorough Write-up with:
Data
Best Fit 4th or 6th degree Polynomial
Theoretical Solution and Graphs
Three Error Analyses
Student Feedback

✔ Students Universally Endorsed the Project / Process. Wished for More.

✔ Interesting Theories on the 6th Degree, Apologies for Poor Data Collection.

✔ Outstanding and Creative Error Estimation

Final Remarks

✔ Success! Students and Faculty Learned and Enjoyed

✔ Preparation and Time Management, Three Class Periods for Two Experiments

✔ Continue and Expand: Two Experiments per Term out of Six or more Scenarios.

✔ Additional Projects: Buckled Column, Discrete Mixture (beans), (frequency) Beats, Spring-Mass System