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STUDENT VERSION
STRUGGLE FOR EXISTENCE

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STATEMENT

We apply an approach to parameter estimation, using two logistic growth models for competing species or strains of one species and historical data from G. F. Gause's work [2] and [3], in the Soviet Union of the 1930's, for studying paramecia and yeast populations.

We point out that Gause was interested in modeling yeast growth (separate species and combined populations) with respect to the production of vodka in state institutions of Mother Russia! In this activity we attack his logistic problem for competition by first determining the (r, K) parameters in separate logistic population models for two yeast species he initially studied. Gause was interested in determining what he called the "coefficients for the struggle for existence," (α and β) in (1) and 2), respectively, which model the intensity of the influence of one species upon the other's carrying capacity, all using his data and our several approaches. We shall leave that for another time.

In (1) and (2) we have the classical competition model found in [4], for example, in which two populations (different species or strains of the same species), N_1 and N_2, with respective growth rates, r_1 and r_2, and carrying capacities, K_1 and K_2, are in the same environment. We shall study this model in more detail later in our studies of differential equations as a modeling tool.

The "coefficients for the struggle for existence" (α and β) in (1) and (2), as Gause called them, were of great interest to him and what he did was to model each species separately (that is EXACTLY what we shall ask you to do) to discover their r and K values and later in our studies, we shall use (1) and (2) to estimate the parameters α and β.

$$\frac{dN_1}{dt} = r_1 N_1 \frac{K_1 - \alpha N_2 - N_1}{K_1} \quad (1)$$

$$\frac{dN_2}{dt} = r_2 N_2 \frac{K_2 - \beta N_1 - N_2}{K_2} \quad (2)$$

	Saccharomyces	Mixed Population	*Schizosaccharomyces*	Mixed Population
Age in hours	Volume of yeast	Volume of yeast	Volume of yeast	Volume of yeast
6	0.37	0.375	-	0.291
16	8.87	3.99	1.00	0.98
24	10.66	4.69	-	1.47
29	12.50	6.15	1.70	1.46
40	13.27	-	-	-
48	12.87	7.27	2.73	1.71
53	12.70	8.30	-	1.84
72	-	-	4.87	-
93	-	-	5.67	-
117	-	-	5.80	-
141	-	-	5.83	-
7.5	1.63	0.923	-	0.371
15.0	6.20	3.082	1.27	0.630
24.0	10.97	5.780	-	1.220
31.5	12.60	9.910	2.33	1.112
33.0	12.90	9.470	-	1.225
44.0	12.77	10.570	-	1.102
51.5	12.90	9.883	4.56	0.961

Table 1. The growth of the yeast volume and the number of cells in pure cultures of *Saccharomyces cerevisiae* (column 1), *Schizosaccharomyces kefir* (column 3) and in the mixed population of these species (column 2 and 4 respectively). [1, p. 395]

In Table 1 we provide the data that Gause obtained from his observations for each population when alone and when mixed in their environment and we will ask you to estimate the respective parameter sets, r_1 and K_1 and then r_2 and K_2, for his two separate populations in the two separate logistics situations (3) and (4).

$$\frac{dN_1}{dt} = r_1 N_1 \frac{K_1 - N_1}{K_1} \tag{3}$$

$$\frac{dN_2}{dt} = r_2 N_2 \frac{K_2 - N_2}{K_2} \tag{4}$$

We need the data for the species in isolation in order to determine their respective r and K values. Table 1 presents Gause's data of two combined "runs" for each species, in isolation and in mixed population.

a) Estimate the respective parameter sets, r_1 and K_1 and then r_2 and K_2, for Gause's two separate populations in the two separate logistics situations (3) and (4) using the appropriate data in Table 2.

We find Gause's estimates for r and K for each of the two population models in his published works (see Table 2) and we include their values below for you to compare your work with his.[1, p. 398]

	Your Analysis		Gause's Analysis	
Species	r	K	r	K
Saccharomyces			0.21827	13.0
Schizosaccharomyces			0.06069	5.8

Table 2. Gause's parameter estimates for the two separate populations of yeast and your estimates for comparison.

b) Using your parameters plot the data and the solution to each of (3) and (4) on the same respective axes. Comment on how good your model is for the data.

c) Using Gause's parameters found in Table 2 plot the data and the solution to each of (3) and (4) on the same respective axes. Comment on how good Gause's model is for the data.

d) Compare your model with Gause's. You should know that he did all of his calculations by hand using a ruler to estimate slopes from the data and then used these slopes to advance his "solutions" for his differential equations. There were no computers to assist in the analyses.

e) Now estimate the parameters α and β and compare your model using (3) and (4) with all your estimated parameters and compare your model result with the data in Table 1.

REFERENCES

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[3] Gause, G. F., O. K. Nastukova, and W. W. Alpatov. 1934. The Influence of Biologically Conditioned Media on the Growth of a Mixed Population of *Paramecium cadatum* and *P. aureliax*. *Journal of Animal Ecology*. 3(2): 222-230.

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