DIFFERENTIAL EQUATIONS AT MANHATTAN COLLEGE PERSONAL ACCOUNT

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Background

The comments here pertain to the 200-level differential equations course that Rosemary Farley taught at Manhattan College (Bronx, NY) in Spring 2016. This course is required of every student in the School of Engineering and some students in the School of Science. As a required course, there is a syllabus with topics that have to be covered in preparation for a common cumulative final. All the traditional methods of solving differential equations by hand must be covered. I taught two of the seven sections and 56 of the 192 students who took the course. All but one of my students was in the School of Engineering. Every student passed the course, however 4 received a grade of D and that grade is not acceptable for this required course in these schools.

This was a 3-credit course that met for two 75-minute sessions each week. One of those weekly sessions was in a computer lab. The differential equations committee chose the textbook [1]together with the online homework system Wiley Plus[5]. In the Spring 2016 semester, the committee added systems of first order differential equations to the curriculum. Linear algebra is not a prerequisite for the course.

Course grades were calculated as follows. There were three in-class examinations, totaling 60%, Maple labs were worth 15%, and a cumulative common final examination was worth 25%. Questions requiring mathematical modeling appeared on all examinations.

The Motivation

My research colleague, Patrice Tiffany, and I attended the MAA-PREP Workshop on SIMIODE at Carroll College in Helena, Montana in the summer of 2015. We left convinced that a modeling first approach should be used in our differential equations classes at Manhattan College. We formed an interesting and symbiotic partnership. I have taught the course for twenty years while she never has taught it. Unfortunately for me, she was on sabbatical in the spring semester when the differential equations courses are primarily taught. Fortunately for me, she was my sounding board for the labs I created in her absence. I benefited from her guidance, as she asked questions and expressed concerns about important issues that escaped my detection. I write this first to give credit where it is due and second to stress that beginning something like this is so much easier if one can work with a colleague.

Getting Started

When thinking about using a modeling first approach, I must admit that I worried about completing the syllabus. At the beginning of the semester, the differential equations committee created a common final that had one question involving modeling. That question had four options and students picked one. The rest of the test looked like most traditional differential equations finals. The topics included first order differential equations, first order systems of two differential equations in two unknowns, second order homogeneous and non-homogeneous differential equations, and Laplace transforms. I wanted to commit to the modeling first approach but I also needed to make sure that my students did well on the final. So I made important decisions from the start.

Since I was in a computer lab on Tuesdays and a traditional classroom on Fridays, I decided to do modeling first activities every Tuesday. Occasionally, I used about 25 minutes of the 75-minute class with a regular lecture. However, I committed 50 minutes of each week–that is 1/3 of my allotted time–to modeling first activities.

I realized that I had to be realistic about covering the material. I did this in two ways. First, I believe that the best decision I made was to use the modeling first approach on required topics. I will discuss this later. Second, I supplemented lectures with videos on the material covered. This was not a flipped classroom. I covered all the material in class and gave more examples and explanations on videos. The creation of these videos was time consuming and I do know that there is a great deal of material available online already. However, I wanted my students to know that I was really committed to the modeling first approach. Many admitted that they placed a higher value on the videos because I made them. The response to the videos was overwhelmingly positive. In fact, I have never had as much gratitude expressed for work done.

I also decided to use the computer algebra system Maple. Some of my students have used Maple in their calculus classes and some have not. I assumed nothing as far as Maple knowledge goes. The code is easily learned and easily forgotten when it is not being used. In fact, there are so many menus now, that Maple is getting easier to use. The learning curve is not steep. I have found that there is little difference between the students who have used Maple before and those who have not. I also believe that the technological tool chosen is not important. However, it is important that differential equations students use technology. The most recent report from the Differential Equations group of the Committee on the Undergraduate Program in Mathematics [2] states that the ordinary differential equations course "is easily the course in the introductory undergraduate mathematics curriculum in which the use of technology is most essential."

Using Technology in the Lab

It is imperative in every lab setting that mathematics takes center stage. From the beginning I de-emphasize code by explaining that when students know what they want to do, I will provide the code for them to do it. There is one rule that helps a great deal. No one is allowed to ask for help with any code until the student has written down what they want to do. Some students will say, "I don't know what to type." Typically what they really mean is "I don't know what to do." It took about two weeks, but finally all students realized that asking for code was not going to help them actually create a differential equation. Rather, they had to interpret the words given about how something like a disease spreads and come up with a differential equation that made sense. Once they understood this, their impatience with Maple diminished and their questions changed, becoming centered on the mathematics in their problem.

Modeling First: Classes in the Lab

On the first day of class, the students were given the M&M Immigration and Death scenario [6] from SIMIODE. This introduction to the modeling first approach was quite successful. Students followed the directions carefully, and arrived at the difference equation quickly. They used the rsolve command in Maple to find a closed form solution and graphed it. All sorts of interesting analyses happened. However, I was disappointed when they struggled to see the emerging calculus ideas as we went from a difference equation to a differential equation. The material was important enough to have a full class discussion about this issue. While it took time, it was a very fruitful and important discussion.

The remaining Tuesday modeling classes followed the same pattern. We did no more data collection. Rather, problems would be posed to the students and their task was to find initial value problems and solve them any way they could. Let me reiterate that the best decision I made was to use the modeling first approach on the required topics. This made me relax about covering material in the syllabus. Before we did any method of solving differential equations by hand, the students had some kind of modeling first activity in the lab. Let me explain further.

The first time students saw Newton's Law of Cooling was in a lab where this law was stated in words. The students were then given a problem and were told that they could use Newton's Law of Cooling to solve it. They had to decide what variables were essential and what to call them. They had to translate the law into a differential equation, identify any initial conditions or important facts, and answer the question. Since they had not learned how to solve any first order differential equations yet, they used Maple. This method of modeling first continued throughout the course. The first time they saw a salt concentration problem with one tank or a spread of disease problem was in the lab. With no lecture beforehand explaining how to proceed, students were asked to solve these problems. As I walked around the room, answering questions, asking questions, and giving advice, students started to speak mathematically. They naturally started to work together, agree on variable names, and discuss what was important. They began to discuss whether or not their answers made sense. They found problems like salt concentration problems relatively easy when they realized that the units could be used to derive the differential equation. They struggled with the spread of disease in a boarding school problem and had to be guided toward creating a differential equation that included the product of those infected and those not infected. Regardless of how hard they found the questions, they were all engaged. There definitely were leaders and followers, but because they each had to hand in a write-up of the solutions, each student made sure to clarify any misunderstandings. For the labs involving first order differential equations, I drew on portions of the SIMIODE scenarios [7, **?**, **?**].

Systems of differential equations were started with a modeling first approach by considering a predator–prey model. Students were told to assume certain facts about how both the prey and predator populations were changing. Then they had to find a system of two differential equations and initial conditions. We talked about the fact that they were really being asked to translate the given assumptions into mathematical equations and that a far more difficult problem would have been to ask them to come up with those assumptions based on observed information. Then the students were asked to open a Maple worksheet that created a system of differential equations that modeled a predator-prey scenario. In this worksheet, the system of differential equations was not solved. Rather, both the direction field and a particular solution for certain initial conditions were plotted. Then they were asked to discuss what this animation revealed in terms of predators and prey. It was interesting to listen as groups talked about what was being revealed on the plot. The students were speaking about how one population depended on the other. It was also great to realize that these differential equations were coming alive as a result of this exercise.

The students were then given a two-tank salt concentration problem. Realize that the first system they had ever seen was the predator-prey system a few minutes before. It was simply fascinating, and a bit unnerving, to watch as students tried to cram every piece of information into one differential equation. The reaction was one of relief as they realized they needed two differential equations! This lab was great. Students saw systems of differential equations from a modeling first perspective. They analyzed their solutions and answered all the subsequent questions from several different perspectives. For the labs involving systems of first order differential equations, consider SIMIODE scenario [9].

To use a modeling first approach when solving second order differential equations, we tackled the spring-mass problem. My goal was to have them derive the second order differential equation that describes such a system. I gave a great deal of guidance, asking specific questions about the forces acting on such a system and about Newton's Second Law of Motion. It turned out to be such a positive experience. The students were all engaged and involved in figuring out how to think about such a system. They were immediately talking about units and about what made sense physically. Ultimately, they arrived at a second order initial value problem of the form: $my''(t) + by'(t) + ky(t) = 0$ with $y(0) = c_1$ and $y'(0) = c_2$. They understood that b was a damping constant and k was a spring constant. However, something interesting happened when they solved subsequent word problems. Many of the students proceeded the way they had derived the formula, rather than just using the formula. It showed me that they really understood the material. I did suggest that on a timed test, they might not derive it again. More than one student said that they liked their way because it involved understanding. This lab was a great success. For the labs involving second order differential equations, consider SIMIODE scenarios [8, **?**].

Reflections and Conclusions

I was far too ambitious on my first labs and by trying to do too much in a lab setting I actually accomplished less. Everything was better when I shortened the labs. I worried too much about time management because I did not realize that the modeling first approach had unforeseen benefits.

For example, when we talked about autonomous differential equations in class and I asked whether we had seen such equations before there was a resounding "Yes." Most students not only knew that the boarding school problem was such an equation but they could describe the graph and its asymptotic behavior. When we did a Newton's Law of Cooling problem in lecture, and I asked what information was given, students called out the correct differential equation and the correct initial conditions immediately. There was little hesitation and their answers were perfect. So we picked up time in lecture because this material could be covered quickly. It was interesting that most of my students approached the modeling problems on tests and the final in way I had not seen previously. In a typical modeling problem, most students wrote down the differential equation, equations for every condition given, and an equation for what they were asked to find before solving anything. This was a direct result of using a computer algebra system to solve differential equations. Having all the information written down before solving anything kept the students on track.

Would I do this again? Absolutely. I will make some changes. I will take time to c heck the SIMIODE website [4] frequently because new scenarios are being added all the time. I had just finished working on my own spring-mass lab in the spring when I found a new one that I could have used. We need not reinvent the wheel. We can use what others have shared. In that vein, I am also so happy that I kept a blog on SIMIODE [3]. It is rather shocking to realize that what I did last semester is already a blur to me. Reading my own blog about my successes, failures, and surprises along the way clarified so many of the ideas reflected in these past few pages. In addition to my blog, every lab I did and all the solutions are also available through the SIMIODE website [**?**]. I wholeheartedly suggest that you take the time to write your own blog and share it with the community of learners in SIMIODE. It is quite a learning experience. It has taken me this one semester to become committed to the modeling first method. This method has definitely helped both me and my students change the way we think about differential equations and make us all better problem solvers.

Given all the modeling my classes did, it was not surprising (but it was gratifying) that my students had the highest scores on the modeling problems on the common final. The actual overall average on this question for all seven sections was 6.80 out of a possible 10. My students' average was 7.63. In fact, my students did very well on the cumulative final as a whole, having an average of 81.8%. That is good news. I really am anxious to teach differential equations again in the Spring of 2017 modifying the approach I used this past semester, having my research colleague back on campus, and benefiting from another several months of the ever-expanding SIMIODE library of scenarios.

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