Logistics of Mathematical Modeling-Focused Projects

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Abstract: Projects provide tangible connections to course content and can motivate students to learn at a deeper level. This article focuses on the implementation of projects in both lower and upper division math courses which develop and analyze mathematical models of a problem based upon known data and real-life situations. Logistical pitfalls and insights are highlighted as well as several key implementation resources. Student feedback demonstrate a positive correlation between the use of projects and an enhanced understanding of the course topics when the impact of logistics is reduced. Best practices learned over the years are given along with example project summaries.

Keywords: modeling, projects, project-based learning, logistics

1 INTRODUCTION

One of my main joys in teaching is helping students draw connections between what they know and what they are learning. I incorporate projects in most of my classes to effectively engage students. Such projects are most naturally done in applied courses, but can be integrated into theoretical courses as well. Positive correlations between student learning and nontraditional teaching techniques such as project-based learning and problem-focused group work in class are shown from the literature in subsection 1.1. My most effective projects have covered the full spectrum of mathematical modeling where students are involved in data collection, processing, developing model equations, and evaluation of their model based upon the results and outside data. When first implementing projects in my classes, my students and I dealt with several logistical issues that needed to be smoothed out for the projects to be beneficial. I share my implementation difficulties, resources I have utilized, and a generalization of these best practices in section 2. Section 3 summarizes specific project implementations, chosen topics, student feedback for classes in which I have implemented major modeling projects. I have found that these best practices depend upon the class size and topic. For example, a project in Differential Equations with 43 students is best spread over 2 weeks in groups of 4, while a project in Advanced Linear Algebra with 14 students is best spread over 5 weeks in groups of 2. Detailed project prompts, class sizes, and grading rubrics can be found in the Appendix.

1.1 Learning Through Projects

Implementing projects in class increases student engagement in class, collaboration outside of class, and new perspectives with which to learn the material. Long-term studies have shown that interactive teaching styles result in significantly higher understanding of concepts [9, 8]. Project-based learning is a teaching methodology that utilizes student-centered projects, often over extended periods of time, to facilitate student learning. Coupled with smaller class sizes (about 40 students), short-term projects have been shown to improve student attitude and achievement [13]. Further, long-term problem-driven approaches in a large class (about 75 students) similarly improves student learning and achievement [16]. Effective collaboration and motivational open-ended questions, two key attributes of project-based learning, are helpful guides for developing class projects. Specifically, group collaboration that involves student choice, communication, writing, revision, and presentation is most effective at increasing student learning [12]. Additionally, group projects provide stimulating discussions and have spurred ideas for individual research projects later on.

Modeling-focused projects are helpful in providing a big-picture perspective, especially at the beginning of a course. For example, on the first day of Mathematical Modeling, I give pairs of students a hazelnut, a jar, and a ruler and ask them to estimate how many hazelnuts fit in the jar. Given 10 minutes, students estimate the number by comparing the volumes of a hazelnut and the jar while ignoring packing loss. After group reflection in class, they are prepared to research packing efficiencies and come back to the next class with much better estimates. This project exemplifies a core principle of project-based learning: students developing mastery and becoming self-directed learners [1]. Additionally, by incorporating parameter estimation from collected data, as in this example, students achieve a more realistic view of the world which helps them utilize their learning later on [18]. Mooney and Swift frame the modeling process as creating an idealized replica of the real world, called the Model World, through simplifying assumptions which are helpful and necessary due to available resources [14]. The solution to the idealized problem is then evaluated in the real world and the model is improved and re-solved as necessary [3]. Starting with a quick visual guess, then volume measurements, and finally accounting for packing efficiency, my students could see the improvement with each successive resolution of their model. This simple activity demonstrates how and why we solve problems in the idealized Model World as well as how many assumptions surround even simple calculations. The room buzzed with anticipation as I counted out the hazelnuts from a packed jar. The way their estimates spread about the exact value (which one student estimated exactly) gave a great segue into statistical measures.

A motivation for these key attributes of the modeling process can be sparked in every project through follow-up questions such as *"Why was this a good way to set up the real question mathematically?," "Is this a good mathematical technique for solving the problem?," "How well can you trust your answer?"*, or *"What do you do with the solution?"* Not knowing how to solve the problem exactly, students may feel overwhelmed with doubt in giving estimated solutions to problems they framed themselves, but encouraging further justification, such as a logical argument developing their model and statistical tests of the data, can help them gain confidence. Facing such open-ended questions in a supportive learning environment helps prepare them for collaborative work in their jobs where they will have to explain and defend their results [18].

2 LOGISTICS

There are many logistical issues that can make implementing projects difficult for both professor and student. These can be minimized through open dialog with students and utilizing available resources.

2.1 Logistical Issues

The main logistical issue for implementing projects is the additional professor workload. Projects are an important way to engage students in a different learning style and encourage positive collaboration. Yet, projects can end up displacing important content and requiring much more time prepare and grade. Further, it feels risky to dedicate an entire class period to group activities.

Other logistical issues revolve around individual student learning. Unequal division of workload is common when group members do not consciously divide up the work. The grade should reflect the distribution of the workload as well as the quality of the project as a whole. As a learning tool, each student needs to know the content of the whole project, not just their individual component. Also, groups who do not plan out their project work end up completing the project at the last minute.

2.2 Resources

I have utilized several different tools in my classes, both in preparing projects and implementing them, to minimize the impact of these logistical issues. Databases of prepared projects and ideas save prep time. Collaboration tools and software lessen the difficulty in student coordination of the project. Course management tools house project work in one place for ease of presenting results, collecting student work, and grading it online.

There are many resources available with outlines and prepared class projects. Toews gives a broad overview of how modeling can be used across the mathematics curriculum [17], while others cater to specific courses such as Calculus [10, 15], Differential Equations [4, 20], Numerical Methods [5], and a course specifically in Mathematical Modeling with emphasis on writing [12]. A repository of modeling activites specifically for Differential Equations called *SIMIODE: Systematic Initiative for Modeling Investigations and Opportunities in Differential Equations* is maintained by a community of educators and directed by Brian Winkel [20]. These activities are peer-reviewed to provide clear instructions for student investigation and instructor facilitation. A similar community-supported repository specifically for Calculus is *Project MOSAIC* [15]. Spurred by the 2013 Mathematics for Planet Earth initiative, the center for Discrete Mathematics and Theoretical Computer Science sponsored the development of several sustainability modules in the same vein as SIMIODE activities applied to Calculus, Differential Equations, Discrete Math, Statistics, and Liberal Arts Mathematics courses [7]. Further, individual projects covering a wide spectrum of topics and classes can be found in the literature through journals such as PRIMUS.

Coordinated file management can help implement projects, even in a small class. A central database for web submission, display, and grading of projects has been a major time saver for me. File submission, forum setup, and wiki creation are supported by most course management systems (my university uses Moodle), and can also be accessed separately through Google Drive (drive.google.com), Drop Box (dropbox.com), and others. A private wiki, accessed through a course management system like Moodle (www.moodle.com) or separately like PBWorks (www.pbworks.com), is a web environment where students can create and link together multiple web pages, similar to Wikipedia in a more controlled environment. Having students complete their project on the wiki keeps everything in one place to make it easier for me to grade. I can also visually check on group work progress and give directed reminders to those trailing behind. I often require students to self-report their individual contributions on the wiki to help them stay accountable and allow me to distribute points according to contribution efficiently. In addition, the fact that students are self-reporting their contributions encourages them to consciously divide up the work evenly at the beginning. The wiki environment also allows for quick transition between oral presentations of multiple groups since they all link back to the page with the project's prompt.

For collaborative data analysis, Google Sheets was effective in providing access to data collection, processing, and visualization. This saved me and the students the hassle of transferring files and made grading easier by keeping all of my students' work in the same document. Because they were all working on the same document, I created a tab for each group to work on and put instructions and example formatting in the first tab. Both the wiki and Google Sheets are handy in group presentations as they decrease the amount of down time transfering files.

To help coordinate group work, I originally encouraged students to use mass emails to myself and their group. Mass emailing is helpful because the emails can be sorted together and a record of the conversation is kept, but it does clutter my email Inbox and email does not transfer mathematical work well. Instead, I now encourage students to post their work on the private wiki which allows posting of figures and mathematical typesetting in HTML. In addition to the benefits mentioned above, the wiki environment allows students to immediately see group updates in the project as a whole, add comments for revisions, and track changes to hold students accountable for contributing their share of the work.

2.3 General Best Practices

Optimal implementation of a project varies by class size and topic, but I have noticed some best practices that apply in general. Incremental notification and implementation of projects is key to merging project work with standard classwork. In preparing your class, seek a good rhythm and balance of skills for placing the project; a very structured project fits best at the beginning and a more open-ended project in the latter part of the course. Embed extensions of lecture content in the project so that class lecture and homework problems feed into the project. Encourage students to better communicate in groups through assigning individual roles and emphasize the need to review each others' work as it relates to the project as a whole. Most of all, include opportunites for students to buy into the project through selecting partners, group role, and sometimes topic. Remind students of their share of the workload and keep them accountable through monitoring individual contributions and, if desired, asking students to evaluate their group.

To minimize prep time it is important to start with just one project that you have tried once yourself, preferably prepared and tested by another source such as those mentioned in subsection 2.2. In working through projects myself, I determine the prerequisite skills and schedule it accordingly. Before I assign a project, I will go over the expectations and demonstrate the technology they will be using whether it is a calculating web applet, wiki environment, or program they must compile themselves. Leading by example helps avoid many technical issues for students.

To keep from suddenly jarring students from lecture mode, I notify them of a project the week before and have them form groups the class period before I assign the project so they are ready to dive in together. How do you get students into groups they like without the groups becoming too lopsided? I started assigning balanced groups myself, but found that students did not work well together. To balance student choice and more diverse groups, I now assign groups from pairs of students where students pair up themselves. Increasing student choice has led to groups bonding quicker and working together more smoothly. Diversifying the groups in terms of major, gender, ethnicity, work ethic, etc. helps students in large classes get to know more of their classmates and break out of isolating cliques. From my past experience, the most successful group sizes were powers of 2, formed from the self-selected couples up to 12 groups per class for reduced grading: 2 for fewer than 24 students, 4 for 24-48 students, and 8 for 48-96 students. At a small private university, my classes range from 7 to 43 with a mean of 34, but earlier on in my career I found groups of 8 (paired groups of 4) worked well for classes of 75 (Pre-Calculus) and 97 (Linear Algebra). In addition, projects two weeks or longer give a thorough team-building experience through coordinated communication and scheduling of group work.

Above all, projects should support student learning in the course as a whole and enhance its breadth, depth, and content accessibility. Encourage students through the project requirements to connect to other disciplines and to reference other researchers' work or data. To add depth, students should be accountable for their own work and communicate it well in writing and verbally. To improve accessibility, students should use technology (such as a Moodle wiki and Google Sheets) in presenting visualizations of their work as well as posting reports online.

3 IMPLEMENTATION

This section gives summaries and student feedback for example projects from three lower division and three upper division mathematics courses: liberal arts mathematics, discrete mathematics, differential equations, numerical methods, mathematical modeling, and advanced linear algebra. For open-ended projects, I keep a list of topics from which I either assign or have students choose. See section 4 for detailed project prompts. These project ideas come primarily from textbook resources or the online community repositories mentioned above: SIMIODE [20] and Project MOSAIC [15]. To broaden simple projects, I often generalize parameters of the model, have students collect or analyze a dataset, or extend the analysis of their solution.

Student feedback can be a helpful evaluative tool, but getting timely and constructive feedback is difficult. To this end, I stagger different types of feedback collection throughout the term. Along with online course evaluations at the end, I offer anonymous midterm evaluations online which guides students' reflection of the course goals and their learning successes and difficulties so far. Project-specific feedback from 2012-2015 course evaluations are grouped into themes in Table 1 with what semester term they occurred more than once while additional course-specific feedback is listed below each project summary. The main

	Fa12	Sp13	Fa13	Sp14	Fa14	Sp15	Fa15
Hindrances							
H1	X						
H2	X	X	X	X			
H3		X	X		X		
Benefits							
B1	X		X	X	X	X	X
B2			X		X	X	X
B3				X	X	X	X

Table 1. Occurrence of Project-Specific Feedback Themes from 2012-2015 (Sp=Spring, Fa=Fall)

themes are grouped by hindrances: (H1) Timing of projects with respect to lecture topics, (H2) Coordinating schedules for group work outside of class, (H3) Technical problems; and benefits: (B1) Abstract topics became more tangible, (B2) Topics made more sense and at a deeper level, (B3) Good rhythm of individual homework and group projects.

Tracking student feedback by semester, I noticed a positive correlation in the volume of comments about the benefits of projects and a negative correlation in the volume of comments about project difficulties. As I repeated classes and had more experience implementing projects, the effect of logistical issues diminished and their benefits increased. This shows that students are less able to benefit from projects when dealing with logistical difficulties, and that the benefits on student learning increase once logistical issues are reduced.

3.1 Differential Equations

Because differential equations are one of the most natural ways to express a dynamical model, this class provides ample opportunity for modeling projects. I assign four 2-week projects in this 15-week course. The first (from SIOMIODE) and fourth (extended from textbook) projects are shown below.

3.1.1 Cold Outbreak Model

- Resources
  - Dorm floor plan; bag of beans
  - Collaborative spreadsheet (Google Sheets)
  - Collaborative private wiki (Moodle)
- Expectations
  - Simulate spread of common cold by shaking beans onto floor plan and tracking infected rooms
  - Develop model and use data to estimate parameters
  - Complete wiki report with 5 minute presentation

Students were enthusiastic about this hands-on activity and appreciated seeing how a differential equation model develops.

3.1.2 Predator-Prey Model

- Resources
  - Collaborative private wiki (Moodle)
  - PPLANE applet (math.rice.edu/∼dfield/dfpp.html)
- Expectations
  - Research qualitative and quantitative data on realistic predator and prey species
  - Develop predator-prey model and estimate parameters
  - Analyze model theoretically and via PPLANE simulation
  - Submit wiki report with 10 minute presentation

Most groups choose standard predator-prey pairs such as orca-sea lion or cheetah-gazelle which have been fun ways to study such animal behavior around the globe. Every semester, however, I have a couple groups with more imaginative selections such as zombies-humans, mutants-humans, and Sith-Jedi. These groups collect data from novels or games on which they base their parameters and compare their results.

While less quantitative in nature, such a project provides a creative outlet for students. After this project in Fall 2013, a student who had been dragging her feet through the class so far exclaimed, "I never knew math could be fun!" It is exciting to see student attitudes impacted so positively by projects.

3.2 Discrete Mathematics

Covering graph theory, analysis of algorithms, and various other topics, discrete mathematics is brimming with applications but scattered over many topics. Thus, I chose one culminating graph theory project on game analysis spanning 4 weeks. In groups of 4, students choose games from a list and develop individual strategies which they test against each other. To space this long-term project, I have groups teach each other their games halfway through the project, before completing their analysis and presenting to the class.

3.2.1 Game Analysis

- Resources
  - Collaborative wiki (Moodle)
- Expectations
  - Choose a board/card game to summarize and analyze
  - Develop and test individual strategies against each other
  - Develop a board state evaluation function and demonstrate it on a limited game tree
  - Complete wiki report and 10 minute presentation

Students shared that this was their favorite part of the course. I also noticed improvement in exam scores after this project.

3.3 Liberal Arts Mathematics

The liberal arts mathematics course at my university, *The World of Mathematics*, emphasizes applications of mathematical concepts in areas such as consumer finance, probability, and statistics. The collaborative spreadsheet can be viewed and copied from the link provided.

3.3.1 Monthly Budget

- Provided Resources
  - Collaborative discussion forum (Moodle)
  - Collaborative spreadsheet for posting information (Google Sheets) docs.google.com/spreadsheets/d/ 17nWc4l2Olfm4AUFGCPViRqd0Ext9-NiCA4MBPm7mg2E
  - Salary statistics (www.salary.com)
  - Cost of living (costofliving.salary.com)
  - Mortgage payments (www.zillow.com)
  - Budget estimates (www.learnvest.com)
  - TEDx talk on budgeting www.tedxwallstreet.com/alexa-von-tobel-one-life-changing-class-you-never-took-2/
- Expectations
  - Collect data for a chosen job, location, and house
  - Complete a budget and compare to the ideal 50-20-30 rule
  - Compare future value of retirement contributions for several initial amounts (simple sensitivity analysis)
  - Choose a best budget from these results and compare within group

Students were surprised at how short their "paid vacation" is in retirement, and overall liked how this project connected course topics to their lives.

3.4 Numerical Methods

After several short technique-learning projects in Numerical Methods, the final 5-week project is open-ended so that students have the flexibility of applying methods covered in class to topics of their choice. Long-term coordination for this project is most easily done in pairs which works well for this upper division class with fewer than 20 students. I set up a schedule of one-on-one meetings and weekly update reports of their project to keep them accountable. Summaries of the schedule and components of this open-ended project are listed below.

3.4.1 Final Project

- Resources
  - Mathematical software (Matlab)
  - Mathematical typesetting software (www.lyx.org)
  - Collaborative forum (Moodle)
- Expectations
  - Choose a topic to analyze numerically
  - (other project objectives)
  - 10-page report typeset in TeX
  - Form handout and 15 minute presentation

Some students use this opportunity to enhance projects in one of their other courses, such as *Simulating a Helical Solenoid* and *Identifying Predictors of Voting Registration*, while others extended topics introduced in class like *Computing Land Area from a Sample of Longitude and Latitude* and *Investigating Fractal Behavior of Various Iteration Functions*.

3.5 Mathematical Modeling

I set up the entire Mathematical Modeling course as a series of 2 week group projects. Though this was a smaller upper division course (12 students), I chose to have groups of 4 to adequately deal with the depth of these projects. Below is a summary of two of these projects, estimating hazelnuts in a jar and modeling the spread of the common cold on campus using an SIR model, both extensions of problems in the textbook. The collaborative spreadsheet can be viewed and copied from the link provided.

3.5.1 Hazelnut Estimation

- Resources
  - Ruler, hazelnut, and jar dimensions
- Expectations
  - Measure volume of hazelnut and estimate how many fit in the given jar
  - Research optimal packing efficiencies and improve initial group estimate
  - Submit report summarizing chosen simplifying assumptions, solution method, and final estimate

Students enjoyed this project as a way to see how many individual decisions and various approaches can go into solving such a simple problem.

3.5.2 Common Cold SIR Model

- Resources
  - Mathematical Software (Matlab)
  - Collaborative Spreadsheet (Google Sheets) docs.google.com/spreadsheets/d/18WqiQb7LU1m-pf_JSP32dEY5VkuCbCqL8aVpqYSEkA8
- Expectations
  - Among the people living near you, track how many of them show signs of a cold every day for a week (40-50 per group)
  - Record data in the spreadsheet and compute average transmission probabilities
  - Form the Markov model with collected transmission probabilities and simulate deterministic and stochastic versions
  - Evaluate coefficient of determination, F-ratio, and t-test statistic for each model
  - Submit report evaluating your model with a 15 minute presentation

Students really enjoyed this chance to collect data and be a part of the dataset themselves.

3.6 Advanced Linear Algebra

For this small (10-20 student) upper division theoretical class, I assign a 5 week project analyzing and extending a research article in groups of 2. This is a simple extension of the modeling process to evaluating and improving upon current (undergraduate-level) research.

3.6.1 Research Article Extension

- Resources
  - Repository of undergraduate-level math research (College Mathematics Journal)
  - Mathematical Typesetting (www.lyx.org)
- Expectations
  - Choose an article on Linear Algebra and replicate its findings
  - Extend this article by generalizing a theorem or applying the given theory in a new way
  - Submit a report summarizing the article and detailing your contribution in extending it.
  - Prepare a 15-minute conference-style presentation

In addition to positive feedback about this project, several of my students have presented their research at a regional conference.

4 CONCLUSIONS

In sum, the best practices I use incorporating projects reduce the impact of logistical issues through incremental notification and guidance of the project. In preparing classes, place a project according to both its prerequisite skills and level of student investigation and embed course content in the project. Encourage students to better communicate in groups through assigning individual roles and emphasize the need to review each others' work as it relates to the project as a whole. Include opportunities for students to buy into the project through selecting partners, group role, and sometimes topic. Remind students of their share of the workload and keep them accountable through monitoring individual contributions. Once the logistical issues are minimized, projects can connect students to course content in new ways and motivate them to learn at a deeper level.

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APPENDIX

This appendix lists the (abridged) project prompts and rubrics (when given) for each of the projects summarized in section 3.

4.1 Liberal Arts Mathematics

Use the given links to estimate monthly expenses as if you were starting a new job today. See collaborative spreadsheet for a template.

1) Estimate monthly salary from salary.com and compute taxes withheld (use 11% Fed +9% State (adjust accordingly)) to determine your take-home pay.

2) Select a home on Zillow to estimate 20% down due and mortgage payments. Note: it is more realistic that you will rent for a while when

you start your job, but it is surprising how quickly you will be interested in buying a house, so it is good to plan ahead for it. How long will it take you to save up enough money for the 20% down payment (you will be charged extra if you do not pay 20% down so this is a good amount to plan for).

3) Check the rate on your student loans and compute the monthly payments needed after you graduate.

4) Choose how much to put aside for retirement. Use the spreadsheet to calculate how much you will have saved up for retirement in 35 years using an average interest rate of 8%, and then divide this number by your starting salary's future value (assuming 3% inflation) to see how many years of living on "paid vacation" you will have earned. Note: your retirement savings may be augmented by Social Security and retirement accounts your employer sets up for you.

5) Estimate other personal expenses (yellow colored cells) using the example values as a starting point.

6) Visualize your distribution of Fixed Costs, Financial Goals, and Flexible Spending. How similar is it to the 50-20-30 rule? In what areas could you trim costs to better fit the rule?

7) Alter the amount put into retirement and compare years of retirement and how it impacts your monthly spending. Choose a best value and explain why it is better than the others you checked. Compare with your group members and discuss your differences.

4.2 Differential Equations

4.3 Cold Outbreak Model

This is an extension of the activity you did on the first day of class. In your same groups of 4 do the following.

A) Record 10 simulations of Brandt Outbreak Model with different number of starting infected in this Google Sheet.

B) Derive differential equation model (with unknown parameters) and find its general solution. Summarize why your model fits. Explain in detail the main steps needed in solving your model. Show the model and solution.

C) For each simulation, estimate the fitting parameter (growth rate) by graphing the general solution with each data set. Show 2-3 example graphs of your best fitting models over the range of initial infected.

D) Summarize all 10 growth rates, their average and range. Show graph of all 10 data sets together. Discuss your results: Was it a good model? Were your parameter values close to the average? How could this model be used? How could this model be improved?

4.3.1 Predator-Prey Model

In groups of 4, choose a dominant predator, a lesser predator, and a common prey to research. We will be using qualitative (graphs) and quantitative (average growth/death rates) data to set up a Predator-Predator-Prey model and analyze the differences between 1 and 2 predators in the system. Create a set of links to (a) describe your chosen scenario with citations, (b) analyze the 2x2 model, and (c) analyze the 3x3 model. Create a group wiki. Give a 10 minute summary of your wiki page and models in class next Friday. Each group member must contribute to both the wiki and the presentation.

Rubric:

10 pts: Wiki setup of Pred-Prey Scenario and citations

10 pts: 2x2 Model Analysis

10 pts: 3x3 Model Analysis

10 pts: Presentation

4.4 Numerical Methods

Choose a topic which can be investigated through a numerical method. Previous topics include electric field of helical solenoid, statistical analysis of voter skepticism, Julian sets (fractals), computing area on a sphere, and heat analysis of air fins.

Planning: Sign up for a 30-minute time slot to meet to nail down a detailed plan for your individual project. Come prepared with your topic and several specific things you would like to investigate about it.

Initialization: Explain your topic, why you are interested in it, and what kinds of numerical methods would be helpful in investigating your topic (named programs I have given you or ones you have written for this class that would be helpful). Outline three objectives for your project: Easy (takes 1-2 hrs), Medium (takes 1 week), Hard (takes >1 weeks).For each objective, outline a program to complete them in its own Matlab file. Upload to foxtale with this report.

Update: For each of your Individual Project objectives, continue the list of what you have accomplished, what you still plan to do, and what part(s) you are stuck on/having trouble with. You should be wrapping things up for your final report next Thursday. List any questions you have for me.

Report: Describe the question you investigated and give some background information on your topic (cite references for all results used). Describe the process used in analyzing the given problem and developing your programs. List all programs used and describe how they are used (including inputs and outputs) Show and describe what each result represents. Then discuss the significance of your results in terms of your investigation and list any avenues that could be interesting to explore further in the future.

Presentation: 10-minute visual presentation of your Final Project. Have a 1-pg flyer printed for everyone in the class. Summarize/demonstrate any code used.

4.5 Mathematical Modeling

4.5.1 Hazelnut Estimation

Estimate the number of hazelnuts that can fit (densely packed) in the jam jar that I brought them in. Jar is modeled as a large lower cylinder with a smaller upper cylinder on top where the edge moves in to fit the lid. Lower cylinder has diameter 2+9/16 in. (6.6 cm) and height 2+3/8 in. (6.5 cm). Upper cylinder has diameter 2+5/16 in (5.9 cm) and height 3/4 in. (1.6 cm).

Use the model representation of each hazelnut of your choice and the measurements your group made last Tuesday to estimate the maximum number of hazelnuts that can fit (without crushing) inside the jar. Share your measurements with your group or you can remeasure a hazelnut in my office (no liquids). Prizes for best estimate(s). Write a summary explaining your assumptions and approximations used in representing the hazelnut.

4.5.2 Cold SIR Model

A) Daily track evidence of having a cold amongst a group living near you for a week. Track 10 people per person, including yourself, and tabulate the number transitioning between states on the Common Cold Spreadsheet

B) Compute average transmission probabilities between states from your sample data, form the Markov model, and write a Matlab file to compute the Deterministic model: X_D.

C) Use these same probabilities to program a Stochastic Markov model, X_S, using the current undergrad population at Fox as your total population.

D) Compute R^2 values for each of your models to your S, I, R group data, "Group_Data". Note, since you just computed averages for your group data (not actual regression), this $R^2 = SS_{Reg}/SS_{Tot}$ actually uses your Markov model populations for $\hat{y}$ not the coefficients. This should be six R^2 values, three comparing to the X_D model and three comparing to the X_S model: for example the first $\hat{y}$ is the deterministic model S in X_D(1,1:n) and y is the S data from Group_Data(1,1:n), where n is the length of your data vector.

E) Use the ColdSIR Regression updated file to compute the estimate Markov probabilities for the Class_Data using multilinear regression. Compare to your group's average values.

F) Compute the F-ratio and t-test statistic of the multilinear regression (b) and evaluate the significance of the trend line and coefficients for the Class_Data.

G) Write up a report and present your findings (10 min) in class.

You can do this inside your Matlab file using %% as section headers and use the Publish command to construct a report with your comments, code, computed results and graphs. See posted ColdSIR Regression file as an example for structuring sections to publish a report.

4.6 Discrete Mathematics

In a group of 4, you will choose a simple game for analysis. Suggestions: Planar Graph Game, Competing Knight's Tour Game, Competing N-Queens Game, Nim, Sim Edge-Coloring Game, Pipe Layer, Mu Torere.

For your chosen simple game, 1) Provide links for further reading and visualizations on this wiki to explain and demonstrate your game to the class.

2) Each person will write up their own strategy (as an algorithm outline) for playing the game in this wiki.

3) Compare strategies through competing multiple times within your group and recording who won the game and in how many moves (or how many possible moves left whichever is easier). Summarize your results on this wiki. Does the first player always win in perfect play or are there conditions on them winning?

4) Lay out a game tree for a 3 move end-game scenario and show how the perfect game is selected. Develop an evaluative function which would be a good heuristic for a fixed depth search and demonstrate it.

5) What mathematical concepts are this game based upon?

4.7 Advanced Linear Algebra

Your Linear Algebra Project will be done in groups of two people and will be presented in class the first week of March. Your project must be based upon an idea presented by a mathematical journal article (e.g. College Mathematics Journal) and must include a piece of original work that you add.

The topics must be relevant to Linear Algebra in a theoretical nature. For example, just using matrices in computation is not enough. Suggested topics: vector space, bases/basis, linear transformation, isomorphism, dual space, eigenvalue/eigenvector, matrix limits, norm, inner product, linear operator, adjoint operator, normal operator, hermitian operator, orthogonal operator, and canonical forms.

Report: Typeset professionally in LyX (or another TeX writer) following the posted template with at least 6 full pages (2 large figures included). Demonstrate your knowledge of and work on a topic chosen from a peer-reviewed research article. Include necessary definitions, theorems with proofs (at least one), and applications/examples to help explain the topic.

Rubric: 100 pts: (25) Typesetting and Organization, (25) Understanding of Background Information, (25) Personal Research Contribution, (25) Follows Outlined Research Proposal

Presentation: Slide presentation in LyX (Beamer class) or Powerpoint or Google Slides. A LyX template with Beamer is posted. Summarize your contributions/work on your chosen topic. Include title page (title/names/professor/school), outline page, Intro/Background, Results, Examples/Applications, Conclusions/Future Work, Citations/Thank you page. Each group member should contribute equally to the oral presentation: (10-15 minutes). Start by introducing yourself. End by thanking the audience. Moderator will ask for questions.

BIOGRAPHICAL SKETCH

R. Corban Harwood earned his Ph.D. in Mathematics from Washington State University in 2011 and is currently Assistant Professor of Mathematics at George Fox University in Newberg, OR. In teaching, he loves drawing connections between mathematics and different disciplines like music, biology, and philosophy. On occasion, Corban does mathematical modeling consulting ranging from testing financial phone apps to designing optimal battery chemistries for hybrid-electric vehicles. His research interests include numerical partial differential equations, semilinear operator splitting methods, and modeling reaction-diffusion phenomena. On the side, he enjoys cycling through the Willamette Valley and hiking to waterfalls with his family.