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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

STUDENT VERSION
Ferris WHEEL CATCH

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STATEMENT

We could attempt the following.

Rendezvous with an Asteroid

The premise of the mission for the spacecraft, Osiris-Rex, is simple: Fly to an asteroid, grab some of the rock and bring it back to Earth, where scientists will study some of the pristine ingredients that went into the making of the solar system, including possibly the building blocks of life.

Once off the ground, Osiris-Rex – a shortening of Origins, Spectral Interpretation, Resource Identification, Security, Regolith Explorer - will be aiming to get close to the asteroid Bennu.

It's 500 meters or so in size, about the height of the Empire State Building.

Osiris-Rex will survey Bennu for more than a year to select the site where it will grab the sample of rock. In July 2020, the spacecraft, about the size of an sport utility vehicle, is to slowly descend and bounce off the surface like a pogo stick at a gentle pace of a quarter-mile per hour. A sampling head, which looks like an automobile air filter, will shoot a burst of nitrogen to kick up dirt and small rocks during the three to five seconds it is in contact with the surface.

The goal is to collect at least a couple of ounces of material and possibly as much as 4.4 pounds. The spacecraft carries enough nitrogen to attempt to extract material three times if necessary.

After departing Bennu in 2021, Osiris-Rex will pass by Earth in September 2023, dropping off a capsule with the samples that will land via parachute in a Utah desert.[1]

But we won't. Rather we shall try something simpler.

Throwing a Bracelet to Someone on a Ferris Wheel

The idea for this activity came from [2] with the title, "Problems that Teach the Obvious but Difficult." We have "shaped" the setting.

You arrived late (as usual) at the amusement park and your girlfriend is already on the Ferris wheel (without you!) You have purchased a bracelet for her (not yet ready for the big engagement ring thing) and you now wish to surprise her and throw it to her as she goes round and round, up and down. You were never one with tact. So with the 20 foot radius Ferris wheel spinning at 5 rpm's and you standing some 60 feet away on the ground, in the plane of the Ferris wheel which is turning towards you, you decide to toss the bracelet to her chair in the hopes of showing off your physical prowess and making up for being late, to say nothing of winning her heart. Did we mention that you never really had tact?

Let us make some assumptions that would permit you to build a differential equation model to determine how you should throw the bracelet to get it to her.

Here are some reasonable assumptions:

a) Ferris wheel continues to rotate at same speed.

b) Girlfriend is a point on the Ferris wheel. Despite your lack of tact you really do see more in her than a point, but this assumption will ease the analyses.

c) You will be throwing the bracelet with a velocity of no more than 70 mph (you were a reasonably good pitcher on your high school baseball team, not great, just reasonable). A major league pitcher can throw a baseball at 100 mph or 88 ft/s, but you're no major league pitcher.

d) You are 6' tall with arms of length 36".

e) The Ferris wheel is turning downward toward you and you are in the plane of the Ferris wheel.

It will be best to set up a coordinate system. We suggest one in which the Ferris wheel base is on the ground at point $(0,0)$ and you are standing at point$(60,0)$. See Figure 1. $(x(t), y(t))$ will be the position of the bracelet at time t.

Let the position of the release of the bracelet come from the tip of your fingers when your arm is 70 degrees off the vertical behind the base of your head which is 5 ft off the ground. This means that the throw will be perpendicular to a line 70 degrees off vertical. And so the launch is at an angle of 110 degrees off the horizontal of the ground from position $(60+3\sin(70^\circ), 5+3\cos(70^\circ))$. See the start of the arrow of direction of throw at the end of your hand in Figure 1.

Activities

1. Model the motion of the Ferris wheel as a function of time.
2. Model the path of the bracelet. You can assume no resistance in the bracelet's flight. We will presume that the acceleration experienced by the bracelet is $\vec{\mathbf{a}} = (0, -g)$ and is due only to

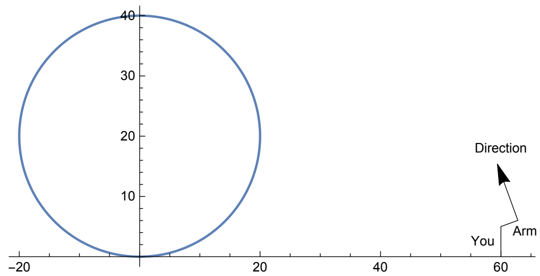

Figure 1. Sketch of launch of bracelet throw to Ferris wheel.

gravity, where g is the acceleration due to gravity and is 32 ft/s^2, i.e. there is no resistance term in our model. This seems reasonable given the nature of the object, a bracelet which has high density and presumably little surface area.

3. Confirm the reasonableness of the release point of the throw of the bracelet toward the Ferris wheel from the above information.
4. Find the range of values of initial speed in which your toss will even come across the path of the Ferris wheel.
5. Pick a reasonable speed at which she will see the bracelet coming toward her. Incidentally, assume she is facing outward and towards where you are standing when the Ferris wheel is coming down. While she is going down on the Ferris wheel she is on the far side facing away from you.
6. Find the exact time you will throw the bracelet so that the bracelet and she arrive at the same time at the same spot.
7. Where is she at the start of your throw and where is she when she grabs the bracelet?
8. What is the relative velocity and speed between her and the bracelet at the instant she catches the bracelet? Is this a reasonable/acceptable speed?

Extensions

Here are some additional notions you might consider.

1. The bracelet should arrive at girlfriends' lap with least relative velocity, i.e. the magnitude of the difference in velocities between the bracelet and the girlfriend. you may assum the mass of the bracelet is 1 slug or 1/32 lb in weight.
2. The bracelet should arrive at girlfriends' lap with least relative kinetic energy, i.e., minimize $1/2mv^2$ where v is the magnitude of the difference in velocities between the bracelet and the girlfriend.
3. The bracelet should arrive at girlfriend's lap as quickly as possible after she begins the Ferris wheel's cycle from the base of the wheel.
4. Consider a resistance term for the flight of the bracelet, something like, $c\vec{\mathbf{v}}(t) = c(x'(t), y'(t))$.

What would be a reasonable value for c. Figure 2 depicts path of bracelet with (red and thick) and without (blue and thin) a resistance term $c\vec{\mathbf{v}}(t) = c(x'(t), y'(t))$ with $c = 0.5$.

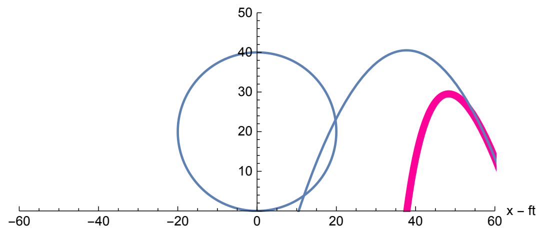

Figure 2. Sketch of two launches of bracelet throw to Ferris wheel with resistance (red, thick plot) and without resistance (thin blue plot).

REFERENCES

[1] Chang, K. 2016. NASA Aims at an Asteroid Holding Clues to the Solar System's Roots. New York Times. 5 September 2016.`http://www.nytimes.com/2016/09/06/science/nasa-osiris-rex-asteroid-bennu-sample.html?_r=0`. Accessed 8 September 2016.

[2] Koblitz, N. 1988. Problems that Teach the Obvious but Difficult. *The American Mathematical Monthly*. 95(3): 254-257.