STATEMENT

We consider a system of linear differential equations

\[\begin{align*}
x'(t) &= ax(t) + by(t) \\
y'(t) &= cx(t) + dy(t)
\end{align*}\]  

with \(x(0) = x_0\) and \(y(0) = y_0\).

We can put this in matrix form:

\[
\begin{bmatrix}
x'(t) \\
y'(t)
\end{bmatrix} = \begin{bmatrix} ax(t) + by(t) \\
cx(t) + dy(t)
\end{bmatrix} = \begin{bmatrix} a & b \\
c & d
\end{bmatrix} \cdot \begin{bmatrix} x(t) \\
y(t)
\end{bmatrix}
\]

with \(x(0) = x_0\) and \(y(0) = y_0\).

OR

\[X'(t) = A \cdot X(t),\]

where

\[
X'(t) = \begin{bmatrix} x'(t) \\
y'(t)
\end{bmatrix}, \quad A = \begin{bmatrix} a & b \\
c & d
\end{bmatrix}, \quad X(t) = \begin{bmatrix} x(t) \\
y(t)
\end{bmatrix}
\]  

\(X'(t) = A \cdot X(t)\) is very suggestive for a solution of the form \(X(t) = c e^{At}\). Thus one could conjecture a solution and try it out to see what \(c\) would have to be, but we have to remember \(A\)
is a matrix, not a number, and while there are theories of what $e^A$ would mean for a matrix $A$ we choose not to go there now.

Instead, we conjecture a vector solution for $X(t)$ which has the best of both worlds (1) nice exponential form with no matrices in the exponent and (2) vector status where we can see the pieces of the solution:

$$X(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} \cdot e^{\lambda t}.$$  

(4)

So here we are suggesting $x(t) = u e^{\lambda t}$ and $y(t) = v e^{\lambda t}$.

**Assignment**

1. Now put the conjecture (4) in (3) and see what has to happen to $a, b, c, d$ and $u, v$ in order to obtain solutions - interesting solutions, i.e. other than $u = 0$ and $v = 0$. Go for it!!! Summarize your conclusions in a nice write-up

2. Illustrate the solution strategy from assignment (1) using the following system

$$\begin{align*}
x'(t) &= -4x(t) - 2y(t) \\
y'(t) &= -1x(t) - 3y(t)
\end{align*}$$  

(5)

with $x(0) = 2$ and $y(0) = 1$. 