## Browsing Your Way to Better Teaching

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**Abstract:** We describe the use of browsing and searching (in libraries, online, inside sources, at meetings, in abstracts, etc.) as a way to stimulate the teacher of undergraduate mathematics, specifically in differential equations. The approach works in all other areas of mathematics. Browsing can help build new and refreshing teaching materials based on how mathematics is used and explored in places other than mathematics. These "other" places are where almost all of our students will be going after they study with us and we should: (i) know about their journey and arrival points; and (ii) understand the disciplinary approaches for those areas which sent these students to us in the first place for their mathematics studies. We describe a personal browsing experience that spanned almost 40 years and proved to be very worthwhile in finding applications of differential equations to modeling Lysergic Acid Diethylamide in the human body.

**Keywords:** Browsing and searching, sources, mathematical modeling, differential equations, compartment model, pharmacokinetics, Lysergic Acid Diethylamide (LSD).

## 1. INTRODUCTION

On American television the National Collegiate Athletic Association runs public service ads in which student athletes say "Most of us will be going pro in something other than sports."[16] This is to counter the image that students who are athletes are all in some chute through college on their way to a professional sports career, but rather will most likely pursue other areas for their livelihood. Furthermore, coaches and those in the college sports scene offer much more than just skill sets for their respective sports. They offer life lessons, examples of how to make decisions, discipline, applications of principles outside sports, etc.

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In the same manner as the student athletes not going pro most of the students who come to our classes to learn mathematics will not go pro, i.e., will not become like us—professional mathematicians or teachers of mathematics. Thus, in addition to sharing the elegance and language of mathematics it behooves us to reach out to where our students are going and do what we can to show them how the mathematics we teach might play a role in their chosen area. We too need to offer life lessons, examples of how to make decisions, discipline, applications of principles outside mathematics, etc.

We discuss one approach to finding mathematical material that would reach out and touch students and help them make the connection to their career path. It is called *browsing*. This is the act of simply looking around you. You do not have to know where you are going. Indeed, Yogi Berra, the great catcher for the New York Yankees Major League Baseball team, expressed the following sage thought [2]:

> If you don't know where you are going, you might wind up someplace else.
>
> You can observe a lot by just watching.

In browsing, ending up "someplace else" might just be the right thing to happen, for you could learn about something really new. The key is to keep your eyes and mind open and see things — you will see a lot! Let new areas register in the foundations of your teaching and explore deeply those which interest you. Then process them and communicate them—to your students as classroom activities and to your colleagues as talks and papers in journals. Do not fail to respond to even more sage advice from Yogi. [2]:

> If you come to a fork in the road, take it.

Some action is in order as you browse. You may not have any agenda other than improving your general understanding of how your mathematics evolved and is useful. So browse with reckless abandon and go where the roads take you.

## 2. PLACES AND SOURCES FOR BROWSING

### 2.1. Collegial Conversation

Whether it is down the hall, at the white board in your office, in a departmental seminar, at a course meeting, or over a coffee break, you have exceptional potential for new ideas and applications through the knowledge base, and willingness to communicate in your day-to-day professional acquaintances. Share experiences, visit each other's classes, and talk about approaches and materials

which will enrich both your classes. Do not hide that which interests your students—share it through internal seminars, talks at meetings, or papers for publication.

## 2.2. Conference and Meeting Presentations

Attending meetings and conferences is an excellent way to acquire teaching ideas and materials involving applications of the mathematics you are teaching. Moreover, you can hear from the initial developer just what has to happen to obtain success for your students: how the presenter got interested in the subject and found the particular materials used in the talk; how much effort it took to transform the ideas from the original source to teaching materials; and what assignments the presenter offered students in this regard.

In lieu of attending events one can often find titles and abstracts of talks with author contact information to initially engage with the presenter and then follow-up on ideas and obtain more specifics, e.g., data sets used for a modeling activity, computer code developed by teacher or student, a student write-up of a related assignment, even more sources that the presenter did not bring to the talk, but which will help you use these ideas in your teaching.

## 2.3. Invited Speakers

Attend seminars in your department as well as those in other departments where they would appear to have some potential for mathematical ideas or would provide general background information to help make you a better browser and teacher. Engage the speaker in questions, stay after the talk to make a contact, and push around some ideas which are floating about in your head due to what you just heard. Act on your curiosity and let yourself browse a myriad of issues that the speaker raised. Do not cut yourself off. Then either let these ideas sit and have your subconscious work on them or act immediately to do a concerted browse to collect specific information and material for building what you want for your own satisfaction and the edification of your students.

## 2.4. Friends and Colleagues Who Know Your Own Interests

Begin to make it known to colleagues that you are interested in learning more about applications of the mathematics you teach or study and that you are interested in acquiring information to help you prepare interesting lessons for your students. You will find that colleagues, who may not be immediately interested in applications of the mathematics, will share through e-mail or send the papers they read.

### 2.5. The Projects or Starred Exercises in the Text

More and more texts are offering project sections in which a model or extended problem is posed, often with supporting data. In the true spirit of scholarship, when these projects are based on the literature there is a set of references offered. Here is where your browsing can begin. The few references cited serve to shape the project, but quite often hold more exploration material and further inquiry for you and your students. Follow the lead and let it take you to even richer material. Of course, the project offered in the text might have natural extensions and explorations offered by the author, but in browsing at the cited articles and beyond you will find a cornucopia of ideas and applications from which you can build your own projects and activities.

### 2.6. The Unbound New Issues of Journals in Your Institution's Library

Most libraries keep the current issues for their journals in a reading room environment so individuals can see the current issue facing out and the other numbers in the current volume laying flat behind the current issue. Thus one can browse a number of issues and their tables of contents. Bounce from article to article. You could prepare for this through browsing of your institution's online journal database in preparation for a library visit or in lieu of it, but then you will miss the touch and feel of the actual journals. Play with the possibilities in these scholarly tomes and take notes on ideas you could use.

### 2.7. Back Issues of Journals on Dusty Library Shelves

In archival sections of libraries there are gems to be discovered and historical approaches to be unearthed, compared, and contrasted with the approach we use today, be it from early 20th century back issues or issues only 10–20 years old. Things change fast, new tools and algorithms come on the scene, and more and more complex problems are tackled. Just a few short years ago such newness added energy in journal dissemination, e.g., catastrophe theory or discrete dynamical systems or social networking. A great deal of material was published in initial bursts, as well as follow-up papers and expository writings. Get a small chair and go to the stacks. You remember doing this as an undergraduate. Immerse yourself in the musty smell of bound journals and browse away.

### 2.8. Older Books in Your Library and Online

Sometimes a current publication will cite a much older book as containing a certain proof or discussion. This older book could be related to some

application that you are working on. Get it from the library and browse through it. Look around the topic of interest for related material and more references you can use to make teaching materials. You can find a number of older, rich source material publications online. A classic work I have used for a number of projects [23, 24] is G. F. Gause's *The Struggle for Existence* [7]. This book is online in English and Russian and has a terrific exposition of population ecology experiments with data on paramecia, didinia, and yeast.

## 2.9. Online Course Descriptions and Courses

Our colleagues are sharing their ideas and approaches to teaching mathematics with applications via their course web pages, where they post the assignments and activities for their students. Certainly, collegial sharing is appropriate when citation is offered as in any academic endeavor. Using teaching ideas learned from others is a time-honored way of increasing the overall effectiveness of the academy.

## 2.10. Newspapers and Magazines

While printed media is shrinking, certainly in its depth of coverage, often the online version of the print source contains very rich sets of reporters' notes, reader reactions and suggestions, and documentation for the article itself. Here you can launch to the writer's sources and find background information of interest to deeper mathematical explorations of interest to you, but perhaps not to the more general reader.

## 2.11. Journals

Both *International Journal of Mathematical Education in Science and Technology* [9] and *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies* [17] focus on undergraduate mathematics instruction, publishing specific and immediately usable material for teaching and developing application activities, as well as offering reflective articles on faculty development and teaching approaches. There are many other journals in this area, as well as discipline-specific journals which reach out to mathematics through articles on interdisciplinary efforts, specific sets of teaching materials and methods, and pedagogical issues in teaching in a discipline which calls upon mathematics. Browse these exceptionally rich sources for teaching ideas.

### 2.12. Professional Society Websites

Professional societies maintain websites which contain or have links to interest groups, blogs, sections, etc. where venturers can find ideas for lessons and activities for teaching. For example, the Mathematical Association of America (MAA) maintains its website [10] to support undergraduate teachers of mathematics and there are regular columns on all aspects of mathematics. One feature of this MAA website is the *MAA Mathematical Sciences Digital Library* [11]. There, under Course Communities, one can find ordinary differential equations and then move to the models section where there are pointers to 21 items including commercial software files, individual faculty web pages, simulation videos, and university courseware sites, to name a few.

### 2.13. Funded Sites

Various government agencies and foundations give support to groups of faculty to produce suitable material on which you can build teaching modules and applications of mathematics. The CODEE Digital Library [5] is one example with lessons, activities, modules, and a journal on teaching differential equations and modeling. Indeed, most funding agencies want to broadcast the successes from their funding and so on their site they will have pointers to programs they have funded with sufficient descriptions so that you will have a sense about whether or not this is a project where you want to browse more intensely.

### 2.14. Online Support Materials

There are a number of web initiatives to support teachers in their efforts to bring more realistic and more applicable projects to their classrooms. COMAP, the Consortium for Mathematics and Its Applications [6] is just one such example. COMAP has a multi-prong approach to sharing project ideas with you. First, there is the set of modules and problems offered by topic area, school level, and degree of difficulty. These come from COMAP's historical module set and more recent problem additions. Second, there is the MATHModels page [13] where a growing number of projects are posted with author commentary and some "solutions"—the latter offered only to teachers and practitioners approved by COMAP. Here you will find projects by topic and difficulty level with attention to prerequisite and topic information. Third, there are two international competitions, the Interdisciplinary Contest in Modeling [8] and the Mathematical Contest in Modeling [12], in which three-person teams of undergraduates attack a modeling scenario over a long weekend and submit their paper for prizes and publication consideration. The backlog of problems

offered over the years and the attendant publications of winning efforts can give you many ideas for projects in your own teaching, be it direct use of a contest problem or some variation, subset, or extension you construct. Finally, there is the *UMAP Journal: Undergraduate Mathematics and Its Applications* [20] in which scholarly articles discuss specific application projects and the general milieux of teaching modeling. The journal promotes itself in this way [20]:

> Each issue puts several real-world problems under a mathematical lens, and demonstrates how real people are using mathematics in their jobs and lives.

### 2.15. Student Assistants

In many colleges and universities the student aid program awards stipends to deserving students in the form of "work-study" grants in which students can work hours at the standard wage on projects the school deems appropriate, up to the amount of the stipend award. This could range in the neighborhood of 10 hours per week for the school year. In the past I have requested such students as graders, journal assistants (for the journals I edited), and "browser" assistants and organizers. In this latter capacity I have set the student out to do some browsing—sometimes quite open and sometimes very directed—for me and for the student, for I believe the students can learn by seeing what I am looking for, namely, applications of mathematics.

### 2.16. Bloggers

Often bloggers have been out there, already exploring new areas of application for the mathematics they are teaching and they want to share their enthusiasm—that is why they blog! Quite often there is discussion of the mathematics, the application context, and the pedagogy with attention to results. Again, going beyond the immediate supplied references can supply you with a rich set of ideas and documented material. Often there is a chance for readers to contribute their feedback so you can see what others have done.

### 2.17. Internet Search Engines

*Google* is the classic "find it" search engine, but others can work for you, e.g., *Yahoo* and *Bing*. One can get lost in the search space, but quite often these tools help us zoom-in on a special area of interest or bring to the fore an area we did not know about through a secondary reference. *Wikipedia* [22] is often cited with some disdain because of the way students use it—their only source! However, the site is rich in information which can serve as a lead

for faculty who are browsing for new teaching ideas and topics, specifically applications of mathematics. Moreover, the material offered is thoroughly referenced and I have found that you can occasionally download the complete pdf of a referenced paper from the *Wikipedia* site.

## 3. A BROWSING RESULT OF INTEREST

In the late 1970s I had a terrific work-study student who served as a Journal Assistant for the journal I edited, but I also asked her to do some browsing in the areas of mathematical biology. She was a biology major and so this fit nicely with her interests. As seminar coordinator at our school I had organized a 3-hour workshop on chemical reaction kinetics modeling by two scientists from The Upjohn Company (formerly a pharmaceutical company, and now split up and in other firms through mergers and acquisitions). They were but an hour away from our campus and when I sought to invite one scientist to speak on the mathematics of modeling and parameter estimation he said he would not come over unless he could bring a chemist and they could do a 3-hour workshop on chemical reactions and parameter estimation. I was reluctant, for I could barely draw a crowd for a 1-hour seminar and this would be a 3-hour workshop. Nevertheless, it worked and we drew a large crowd, some 50 faculty and students. They worked in mixed teams on problems with real data and while the technical material was great, the interactions were even better, for we began to understand what others cared about in their respective fields, chemistry, biology, mathematics, etc.

The lead for the Upjohn visit was Carl Metzler, a statistician and modeler at Upjohn, and it was his work that intrigued me. He was then developing a software package called NONLIN [15] used to estimate parameters in pharmacokinetics models of chemical and biochemical reactions for understanding the actions and timing in drug therapy. When I had my browser student assistant look into other papers in this area she came up with two very interesting ones. The first paper [21] was about the use of NONLIN to estimate parameters in a compartment model of intravenous administration of Lysergic Acid Diethylamide (LSD) to five male volunteers. Indeed, it negatively correlated (correlation coefficient $r = -0.94$) very well the concentration of the LSD in body "tissues" with "performance scores on arithmetic tests" given as the drug's concentration rose and then declined. Later, Metzler offered up a very nice paper [14] with more details, including the actual data on the five volunteers, which he had obtained from the authors of [21] in private correspondence, namely plasma concentration of LSD over an 8-hour period and concurrent test scores.

I was set to dig in to this material and to produce a working modeling activity for my students in my own differential equations class, for I use actual applications in my mathematics classes to motivate as often as I can. However,

I never got to write-up this activity back then. The two papers languished in my files. I carted them around to two schools after that first institution where I taught and only a few weeks ago rediscovered them when moving a huge filing cabinet from the attic to my home office. Lo and behold, there they were on the top shelf. Apparently, I had some papers which were just thrown in drawers and not placed carefully in the more organized file folder in alphabetical order. Thus, it was fortuitous that I found these right away. I also found several other very interesting papers which will form the basis of other work, but I have not gone back to moving the file cabinet yet as the mathematics and its application have consumed me, as much of it did throughout my career.

I want to share this project idea in the hope that you might give it a try in your own teaching and enjoy the buried applications of mathematics which I found years ago through browsing with the help of a student assistant browser.

## 4.  MODELING THE PHARMACOKINETICS OF LSD: A COMPARTMENT MODEL WITH PARAMETER ESTIMATION

In the 1960s there was quite a bit of interest in the drug LSD, *much* interest, from Timothy Leary [4], to government intelligence organizations [18], to scientists [21], to individuals, to reporters writing about the 1960's scene. They say that if you can remember the 1960s then you did not live the 1960s, for if you could remember then you were not stoned enough on drugs and were pretty square. Well, I was square and did not even inhale [3], and I did not have an interest in academic papers in applied mathematics or LSD research back in the 1960s, for I was doing my graduate work in Noetherian rings and that kept me "clean." So I missed the papers described below and did not first come upon them until browsing in the late 1970s.

One of the issues surrounding the use of LSD was what it did to mind performance—enhance, detract, alter? Thus, there were a number of studies conducted to measure the effects of LSD on mental processes. One study [21] attempted to see if there was a correlation between a subject's ability to perform simple arithmetic tasks and the concentration of LSD in the tissues of the body. In an earlier study [1] five normal male volunteer subjects, ages 21 to 25, were administered 2 mcg (ng) of LSD per kilogram of body mass doses of d-Lysergic Acid Diethylamide (LSD-25) intravenously over a 1.5 minute period. Blood samples were then drawn at 5, 15, 30, 60, 120, 240, and 480 minutes and these were tested for concentration levels of LSD.

> To obtain a crude index of performance, subjects were given one of a series of equivalent tests, consisting of simple addition problems, after each blood sample was drawn. [1, p. 612]

This information is given in Table 1.

**Table 1.** Summary of data collected [1, 14] on five male volunteers who were administered LSD and then tested on performance (Perform score (%)) on simple addition questions. Both performance score and plasma concentrations of LSD were recorded at 5, 15, 30, 60, 120, 240, and 480 minutes after the initial infusion of LSD

| | | Time (hour) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 0.833 | 0.25 | 0.5 | 1.0 | 2.0 | 4.0 | 8.0 |
| Subject 1 | Plasma conc. (ng/ml) | 11.1 | 7.4 | 6.3 | 6.9 | 5. | 3.1 | 0.8 |
| | Perform score (%) | 73 | 60 | 35 | 50 | 48 | 73 | 97 |
| Subject 2 | Plasma conc. (ng/ml) | 10.6 | 7.6 | 7. | 4.8 | 2.8 | 2.5 | 2. |
| | Perform score (%) | 72 | 55 | 74 | 81 | 79 | 89 | 106 |
| Subject 3 | Plasma conc. (ng/ml) | 8.7 | 6.7 | 5.9 | 4.3 | 4.4 | − | 0.3 |
| | Perform score (%) | 60 | 23 | 6 | 0 | 27 | 69 | 81 |
| Subject 4 | Plasma conc. (ng/ml) | 10.9 | 8.2 | 7.9 | 6.6 | 5.3 | 3.8 | 1.2 |
| | Perform score (%) | 60 | 20 | 3 | 5 | 3 | 20 | 62 |
| Subject 5 | Plasma conc. (ng/ml) | 6.4 | 6.3 | 5.1 | 4.3 | 3.4 | 1.9 | 0.7 |
| | Perform score (%) | 78 | 65 | 27 | 30 | 35 | 43 | 51 |

### 4.1. Model Building Using Compartment Models

The papers [14, 21] use a simple compartment model in which there are two compartments, Plasma (inner) compartment and tissue (outer) compartment. See Figure 1. Neither paper gives the form of the model using differential equations. Rather, [21] gives the closed-form analytic solution for concentrations of LSD in the two compartment that one can obtain from the linear system of first-order differential equations for $C_P(t)$ (ng/ml), the concentration of LSD in the plasma compartment, and $C_T(t)$ (ng/ml), the concentration of LSD in the tissue compartment. The paper [14] refers to these results.

We are in the business of teaching mathematical modeling so we always take our students back to the basics (How much change in the amount of stuff is there at some place at time $t$?), both mathematically and with regard to the discipline under study, in this case chemical reactions.

We set out the diagram in Figure 1 for students to consider or we have them build it generically with some narrative provided so they can see the appropriate form and information needed to get a picture, which then leads to differential equation models.

This figuring out of units and initial concentration is sometimes a difficult bookkeeping activity for students, but it is essential if we are to proceed and be good applied mathematicians. The conversion equation (1) will help students.[1]

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[1]Thanks to a careful referee read we point out that the "density of human plasma (and human tissue) is about 1 kg/l." Thus, we use, where appropriate, either mass in kilograms or volume in liters, equivalently, as our measure.
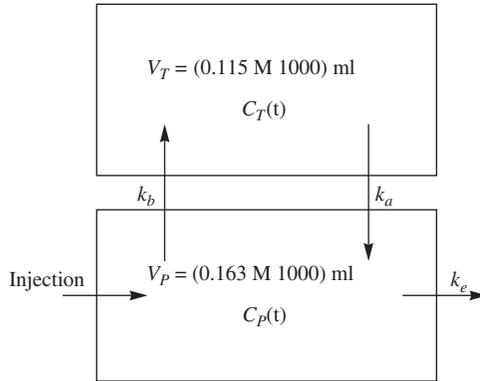
*Figure 1.* Two compartment model for LSD flow between plasma and tissue compartments in the human body where the volume (in milliliters) of the plasma ($V_P$) is 16.3% of the volume or mass ($M$) of the body and the volume of the tissue ($V_T$) portion of the body is 11.5% of the volume or mass of the body.

$$V_P = \underbrace{(0.163M)}_{\text{kg of plasma}} \times \underbrace{(1)}_{\text{liter/kg}} \times \underbrace{(1000)}_{\text{ml/liter}} . = \underbrace{(163M)}_{\text{ml of plasma}} \qquad (1)$$

Using the notion of "simple change in something," in this case amount of LSD in each compartment we can produce the system of differential equations in equation (2). We discourage them from building rate of change models of just concentration as they can be difficult with units. The last term in the equation for $C'_T(t)$ in equation (2) reflects the exponential decay of the LSD in the tissue compartment due to excretion,

$$V_P C'_P(t) = k_a V_T C_T(t) - k_b V_P C_P(t) - k_e V_P C_P(t)$$

$$V_T C'_T(t) = k_b V_P C_P(t) - k_a V_T C_T(t). \qquad (2)$$

We note that $C_P(0) = 12.2699$, for originally LSD was injected at a concentration of 2000 ng/kg of body mass for each subject and so we have an initial concentration in the plasma of $12.2699 = 2000M/.163M \, 1000$ ng of LSD/kg of body mass. Since no LSD is in the tissues at the start (we presume!), $C_T(0) = 0$. Also, we have $V_P = (0.163M \times 1000)$ ml and $V_T = (0.115M \times 1000)$ ml, where $M$ is the subjects' body mass in kilograms. For the mass devoted to the plasma in a normal body is about 16.3% while the mass devoted to the tissues is about 11.5%.

We need to estimate the rate term parameters $k_a$, $k_b$, and $k_e$, each with units $1/hour$ if we are to fully understand this process. In the literature [21] the average set of concentrations for all five subjects at each time, respectively, is used to fit the solutions using the least square criterion in

which we square the difference between the analytic solution for $C_P(t_i)$ and the actual average concentration of LSD (ng/kg) in the plasma for $t = 0.833, 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$ hours. Of course we could take individual subjects and estimate their respective parameters $k_a$, $k_b$, and $k_e$, and we have done that procedure.

So how do students estimate parameters? They use *Mathematica* to get closed-form solutions for $C_P(t)$ and $C_T(t)$, each of which is about five lines of complex algebraic expressions in the familiar format of solutions to such systems, i.e., sums of exponentials with messy terms involving $k_a$, $k_b$, and $k_e$. If we place the average values (for all five subjects) of the concentration of LSD in the plasma at each time $t = 0.8333, 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$ into an array, calling it $a$, with time as the first coordinate and this concentration of LSD in the plasma at the corresponding time as the second coordinate we have seven (from observations) time–concentration data pairs and from these we form the sum of square errors (3) between the model $C_P(t)$ values and the observed values for concentration of LSD in the plasma. Call the latter $O_1$, $i = 1, 2, \ldots, 7$:

$$SSE(k_a, k_b, k_e) = \sum_{i=1}^{7} (C_P(t_i) - O_i)^2. \qquad (3)$$

Now with *Mathematica*'s powerful `FindMinimum` command we can determine the values of the parameters $k_a$, $k_b$, and $k_e$ which minimize this $SSE(k_a, k_b, k_e)$ function:

`FindMinimum[SSE[`$k_a$`, `$k_b$`, `$k_e$`], {`$k_a$`, 1/3}, {`$k_b$`, 1/4}, {`$k_e$`, 1/4}]`

Indeed, we obtain the minimum sum of square errors to be 0.080945, when $k_a = 4.63679$, $k_b = 3.18659$, and $k_e = 0.41128$. We encourage students to use different initial guesses in the `FindMinimum` command for each of the three parameters, $k_a$, $k_b$, and $k_e$, to give them some idea of the robustness of the command itself and confidence that they have a true minimum sum of square errors. This gives us a final model expression for $C_P(t)$ in equation (4):

$$C_P(t) = 0.128,905 \left( 41.2194 e^{-7.99617t} + 53.9669 e^{-0.238492t} \right). \qquad (4)$$

Figure 2 shows a plot of our model (solid line) over the observed average values of concentration (squares) for our five subjects. This is a very convincing plot for students to validate the model and the estimated parameters. Moreover, when they examine the source papers which I can provide them (most libraries do not get the *Clinical Pharmacology and Therapeutics* journal as it is a clinical, not just scientific, publication) their values for the parameters are about the same values found there using primitive techniques of breaking up curves into regions and estimating parameters individually with linear fits after logging

Plasma Conc. LSD 25 – ng/ml
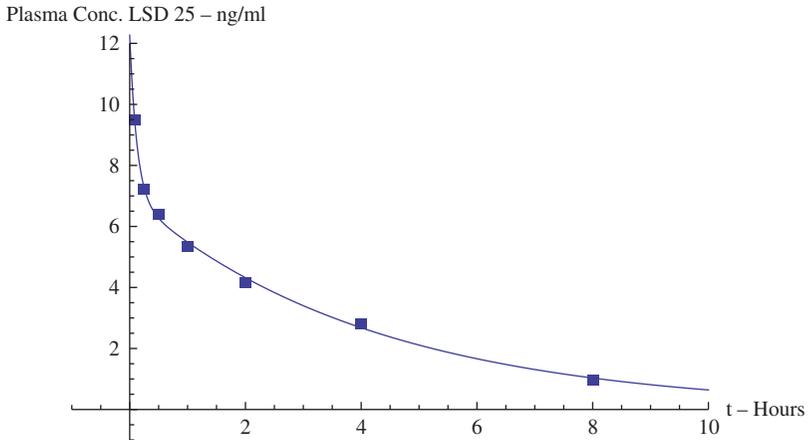


***Figure 2.*** Plot of the observed values of the average concentration of LSD (ng/ml) (squares) and the model built from parameter ($k_a$, $k_b$, and $k_e$) estimates using the solution of the system of differential equations (2) and minimization of the sum of square error function (3) (color figure available online).

the data if necessary. This is in the time-honored manner of how kineticists estimate parameters.

If we take the best fitting parameter values and put them into the complicated expression for $C_T(t)$, the concentration of LSD in ng/ml in the tissue, then we have a model for the LSD presence in tissue. The final model expression for $C_T(t)$ is in equation (5):
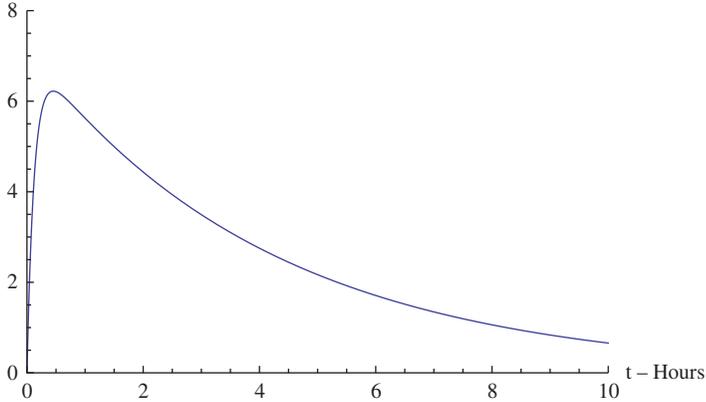
$$C_T(t) = 0.128905 \left( 55.419 e^{-0.238492t} - 55.419 e^{-7.99617t} \right). \tag{5}$$

Figure 3 shows the model prediction of the concentration of LSD in ng/ml in the tissue compartment given our model and the average of the observed subject values for the concentration of LSD in ng/ml in the plasma compartment. This model passes the students' reality check and certainly substantiates the notion of a "high" from drug use.

## 5. ADDED ATTRACTION TO THE LSD MODEL

Recall, we mentioned the effort in [21] to correlate performance test scores with "tissue concentration" of LSD. In Table 1 we find performance scores on simple arithmetic tests conducted at the same time as the drawing of plasma for determining the concentration in ng/ml of LSD in the plasma compartment. Figure 4 shows the relationship between Performance Score (%) (*PS*) on the simple arithmetic problems versus the model prediction of the concentration of

Tissue Conc. LSD 25 – ng/ml



**Figure 3.** Plot of the model of tissue concentration of LSD in ng/ml. This model is built from parameter ($k_a$, $k_b$, and $k_e$) estimates using the solution of the system of differential equations (2) and minimization of the sum of square error function (3) (color figure available online).

Math Test Score %
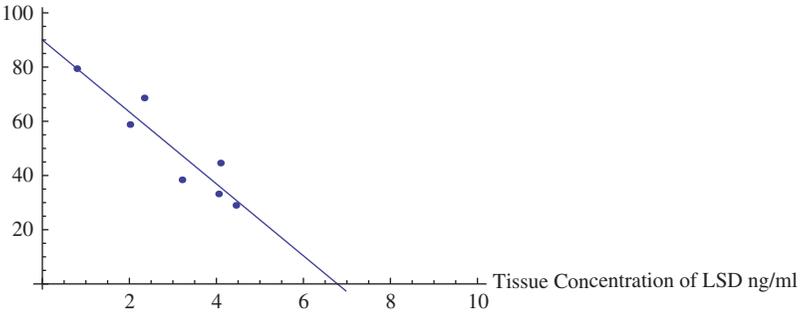


**Figure 4.** Plot of performance score (%) (*PS*) on the simple arithmetic problems versus the model prediction of the concentration of LSD in ng/ml in the tissue compartment (*CT*). Superimposed is the best line of regression whose equation is $PS = 89.1729 - 9.32941CT$. This means for every ng/ml increase in LSD in the tissue compartment the score drops a little over nine points (color figure available online).

LSD in ng/ml in the tissue compartment (*CT*) along with the line of regression. It looks like there is a good correlation. Indeed, the correlation is –0.922682 which is quite good. This final aspect of the two papers we browsed for is of interest to students who are informed in the social sciences and it demonstrates the power of differential equation models in many disciplines.

We do not spend much time on statistical issues [25] when focusing on the differential equation modeling, for we cannot do everything. However, we point out some issues concerning how we determine the parameters, how confident

we are in our estimates, and how these parameters were determined historically [19]. As to the latter point we illustrate how the three regions (absorption—initial steep rise, peak—high point, and elimination—taper) of the tissue concentration curve in Figure 3 were used to estimate various parameters through linearizing with logarithms the function in these regions to get rough estimates of the various parameters, $k_a$, $k_b$, and $k_e$, contrasting this with our one fell swoop approach in *Mathematica* with the `FindMinimum` command.

## 6. CONCLUSION

This entire activity would take one or two periods and an outside writing assignment with a significant write-up of both the mathematics and the modeling work. I have used this sort of modeling activity before studying systems of differential equations to introduce systems thinking and I have used it in the midst of studying systems. Either way, modeling activities can serve to motivate the study. I have found that once we have done a few models with parameter estimation students pick up on this quickly and see the big picture as well as the use of *Mathematica* quite easily. This is not a semester-project-type problem, rather students enjoy seeing their differential equations modeling skills pay off in yet another area just as they study the mathematics; all because the professor went browsing!

I believe it is important to motivate the study of differential equations. The appropriate and historical way I see to do this is through applications in which the differential equations arise naturally as a model for the phenomena. Furthermore, I believe it is important, not to dwell on all the by-hand techniques of the past, but to use technology such as *Mathematica* or *Maple* so the students can see the structures (however complicated they may be) of the solutions and analyze the data. This LSD study illustrates just how this comes together and serves as an example of the power of browsing.

## ACKNOWLEDGEMENT AND ADDITIONAL SOURCES

A referee who was very helpful in improving the exposition of the paper suggested additional good idea sources. We quote:

There is a kind of reading I have found to be helpful which sort of lies between the "newspaper/magazines" and "journals" categories. Every day I take a few minutes to browse some online popular science magazines. The online versions of these publications are updated daily. Most days they just give me a jolt of enthusiasm and stimulate my curiosity, but every so often I find something that could be of interest to students. When that happens, the articles usually have links and references that flesh out the technical details enough to determine whether an associated lesson could be designed for the classroom. Some sites

I visit are *Phys.Org* (www.phys.org), *Science Daily* (www.sciencedaily.com), *Science News* (www.sciencenews.org), and *New Scientist* (www.newscientist.com/section/science-news). Of course there are many others.

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## BIOGRAPHICAL SKETCH

Since moving to Emeritus status in May 2011 at the United States Military Academy at West Point, Brian Winkel has enjoyed the attendant freedom to pursue his own interests at his own pace. This has led to revisits to and publishing of previously discovered teaching approaches. While he misses his former role as mentor to younger faculty through personal contact he realizes the potential to reach colleagues through sharing and writing up his ideas. His background is in abstract algebraic inquiry—quotient rings of Noetherian rings to be specific, but over the years (many) he has enjoyed discovering and teaching applications of mathematics. He is the founder and Editor Emeritus of the journal, *PRIMUS – Problems, Resources, and Issues in Mathematics Undergraduate Studies*, and also founder and Editor Emeritus of the journal *Cryptologia*.